CE 416

Prestressed Concrete Sessional

(Lab Manual)


Department of Civil Engineering

Ahsanullah University of Science and Technology

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Preface

The idea of prestressed concrete has been developed around the latter decades of the 19th century, but its use was limited by the quality of the materials at that time. It took until the 1920s and ’30s for its materials development to progress to a level where prestressed concrete could be used with confidence. Currently many bridges and skyscrapers are designed as prestressed structures. This manual intends to provide a general overview about the design procedure of a two way post tensioned slab and a girder. To provide a complete idea, the stress computation, the reinforcement detailing, shear design, the jacking procedure etc. are discussed in details.

This Lab manual was prepared with the help of the renowned text book "Design of Prestressed Concrete Structures", 3rd Edition by T.Y. Lin and Ned H. Burns. The design steps for a two-way post-tensioned slab was prepared according to the simple hand calculation provided by PCA (Portland Cement Association), ACI 318-05 and BNBC 2020 code requirements. The design steps for a post-tensioned composite bridge girder were prepared with the help of several sample design calculations demonstrated in different PC structure design books and seminar papers. It has been done in accordance with AASHTO LRFD Bridge Design Specifications.

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INDEX

1. INTRODUCTION

Prestressed concrete is a method for overcoming concrete's natural weakness in tension. It can be used to produce beams, floors or bridges with a longer span than is practical with ordinary reinforced concrete. Prestressing tendons (generally of high tensile steel cable or rods) are used to provide a clamping load which produces a compressive stress that balances the tensile stress that the concrete compression member would otherwise experience due to a bending load. Traditional reinforced concrete is based on the use of steel reinforcement bars, rebars, inside poured concrete.

Prestressing can be accomplished in three ways:

- Pre-tensioned concrete,
- Bonded or
- Unbonded post-tensioned concrete.

Pre-tensioned concrete

Pre-tensioned concrete is cast around already tensioned tendons. This method produces a good bond between the tendon and concrete, which both protects the tendon from corrosion and allows for direct transfer of tension. The cured concrete adheres and bonds to the bars and when the tension is released it is transferred to the concrete as compression by static friction. However, it requires stout anchoring points between which the tendon is to be stretched and the tendons are usually in a straight line. Thus, most pre-tensioned concrete elements are prefabricated in a factory and must be transported to the construction site, which limits their size. Pre- tensioned elements may be balcony elements, lintels, floor slabs, beams or foundation piles.

Bonded post-tensioned concrete

Bonded post-tensioned concrete is the descriptive term for a method of applying compression after pouring concrete and the curing process (*in situ*). The concrete is cast around a plastic, steel or aluminium curved duct, to follow the area where otherwise tension would occur in the concrete element. A set of tendons are fished through the duct and the concrete is poured. Once the concrete has hardened, the tendons are tensioned by hydraulic jacks that react against the concrete member itself. When the tendons have stretched sufficiently, according to the design specifications (see Hooke's law), they are wedged in position and maintain tension after the jacks are removed, transferring pressure to the concrete. The duct is then grouted to protect the tendons from corrosion. This method is commonly used to create monolithic slabs for house construction in locations where expansive soils (such as adobe clay) create problems for the typical perimeter foundation. All stresses from seasonal expansion and contraction of the underlying soil are taken into the entire tensioned slab, which supports the building without significant flexure. Post-tensioning is also used in the construction of various bridges, both after concrete is cured after support by falsework and by the assembly of prefabricated sections, as in the segmental bridge. The advantages of this system over un-bonded post-tensioning are:

1. Large reduction in traditional reinforcement requirements as tendons cannot distress in accidents.
2. Tendons can be easily 'weaved' allowing a more efficient design approach.
3. Higher ultimate strength due to bond generated between the strand and concrete.
4. No long term issues with maintaining the integrity of the anchor/dead end.

Un-bonded post-tensioned concrete

Un-bonded post-tensioned concrete differs from bonded post-tensioning by providing each individual cable permanent freedom of movement relative to the concrete. To achieve this, each individual tendon is coated with a grease (generally lithium based) and covered by a plastic sheathing formed in an extrusion process. The transfer of tension to the concrete is achieved by the steel cable acting against steel anchors embedded in the perimeter of the slab. The main disadvantage over bonded post-tensioning is the fact that a cable can distress itself and burst out of the slab if damaged (such as during repair on the slab). The advantages of this system over bonded post-tensioning are:

1. The ability to individually adjust cables based on poor field conditions (For example: shifting a group of 4 cables around an opening by placing 2 to either side).
2. The procedure of post-stress grouting is eliminated.
3. The ability to de-stress the tendons before attempting repair work.

Applications:

- Prestressed concrete is the predominating material for floors in high-rise buildings and the entire containment vessels of nuclear reactors.
- Un-bonded post-tensioning tendons are commonly used in parking garages as barrier cable. Also, due to its ability to be stressed and then de- stressed, it can be used to temporarily repair a damaged building by holding up a damaged wall or floor until permanent repairs can be made.
- The advantages of prestressed concrete include crack control and lower construction costs; thinner slabs - especially important in high rise buildings in which floor thickness savings can translate into additional floors for the same (or lower) cost and fewer joints, since the distance that can be spanned by post-

tensioned slabs exceeds that of reinforced constructions with the same thickness. Increasing span lengths increases the usable unencumbered floorspace in buildings; diminishing the number of joints leads to lower maintenance costs over the design life of a building, since joints are the major focus of weakness in concrete buildings.

- The first prestressed concrete bridge in North America was the Walnut Lane Memorial Bridge in Philadelphia, Pennsylvania. It was completed and opened to traffic in 1951. Prestressing can also be accomplished on circular concrete pipes used for water transmission. High tensile strength steel wire is helically-wrapped around the outside of the pipe under controlled tension and spacing which induces a circumferential compressive stress in the core concrete. This enables the pipe to handle high internal pressures and the effects of external earth and traffic loads.

Design Example of a Post-Tensioned Composite Bridge Girder

General

This chapter demonstrates the detailed analysis & design of a 40m simple span Pre-stressed Post-Tensioned I/Bulb Tee Girder. An exterior girder of a double lane bridge having total width of 9.8 m and carriage width of 7.3 m is considered as per our national standard of double lane highway. The design follows AASHTO LRFD Bridge Design Specifications 2012.

(All dimensions are in mm unless otherwise stated)

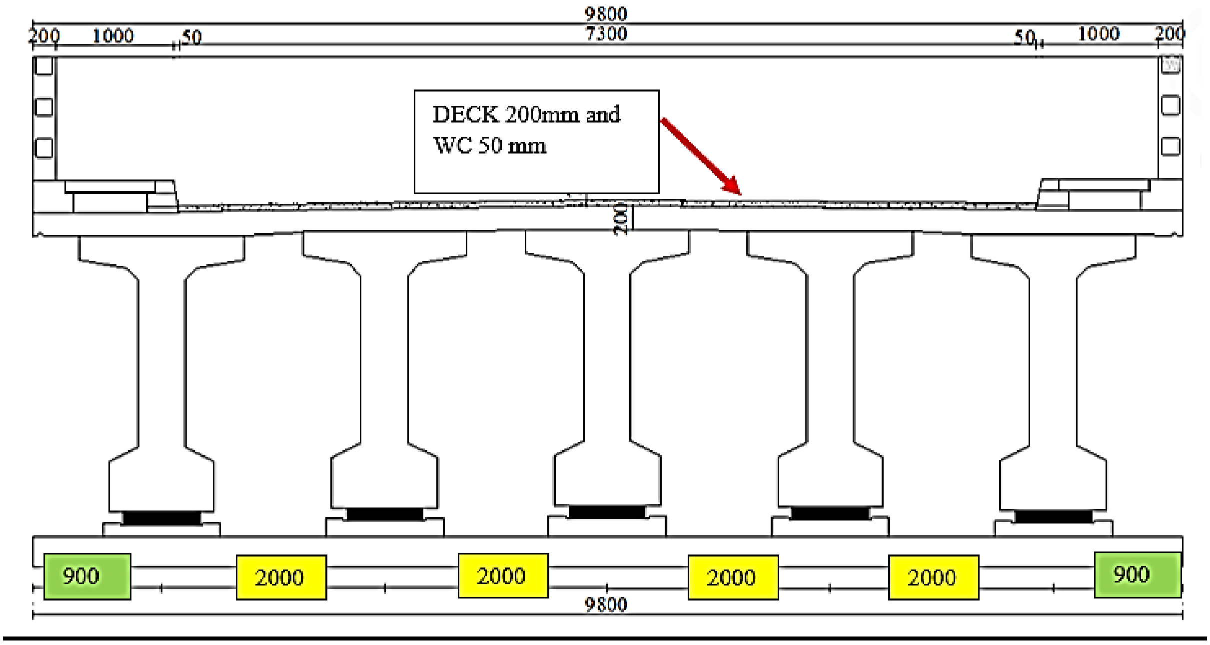


Figure 1: Cross-Section of Deck Slab and Girder

Specifications

Girder Details

Girder Location	=	Exterior Girder
Girder Type	=	Post-Tensioned Pre-stressed I/Bulb Tee Girder
Overall Span Length	=	40 m
CL of Bearing	=	0.45 m
Effective Span Length	=	39.1 m
Girder Depth without Slab	=	0.045 x 40 – 0.2 (Deck) = 1.6 m [AASTHO LRFD `12, Table 2.5.2.6.3-1]
Nos. of Girder	=	5
Spacing of Main Girder	=	2 m (interior girder) [AASHTO LFRD Bridge Construction Specifications, 4th Edition]
	=	1.9 m (exterior girder)

Deck Slab

Thickness	=	0.2 m [AASTHO LRFD `12, Art 9.7.1.1]
Total width	=	9.8 m
Carriage way	=	7.3 m
Nos. of Lane	=	2
Thickness of WC	=	0.05 m

Cross Girder

Number of Cross Girder	=	5 Nos. (2 exterior and 3 interior) (Usually given at 8m to 12m interval)
Depth	=	1.1 m
Thickness of Interior Cross Girder	=	0.3 m (General practice for interior cross girder)
Thickness of Exterior Cross Girder	=	0.5 m (From design, to ensure changing of bearing pad)

Concrete Material Properties

Strength of Girder Concrete, f'_c	=	40MPa [28 days Cylinder Strength]
Strength at Transfer, f'_{ci}	=	40x90% = 36 MPa [21 days]
Strength of Deck Slab	=	30MPa [28 days Cylinder Strength]
Unit Weight of Concrete	=	24 KN/m^3
Unit Weight of WC	=	23 KN/m^3
MOE of Girder Concrete, E_c	=	$0.043K_1{\gamma_c}^{1.5}\sqrt{f'_c}$ = $0.043 * 1 * 24^{1.5} * \sqrt{40} * 1000$ = 31975 MPa [AASTHO LRFD `12, Art 5.4.2.4-1]
MOE of Girder Concrete at Transfer, E_{ci}	=	$0.043K_1{\gamma_c}^{1.5}\sqrt{f'_c}$ = $0.043 * 1 * 24^{1.5} * \sqrt{36} * 1000$ = 30335 MPa
MOE of Deck Slab Concrete, E_s	=	$0.043K_1{\gamma_c}^{1.5}\sqrt{f'_c}$ = $0.043 * 1 * 24^{1.5} * \sqrt{30} * 1000$ = 27690 MPa

Prestressing Material Properties

Anchorage Type	=	19K15 [k-series anchorage]
Strand Details	=	[15.24 mm dia. 7 Ply low relaxation]
Nos. of Strand	=	19
Ultimate Strength of Strand, f_{pu}	=	1860 MPa [AASTHO LRFD `12, Table 5.4.4.1-1]
Yield Strength, f_y	=	0.9 f_{pu}= 1674 MPa [AASTHO LRFD `12, Table 5.4.4.1-1]
MOE of Strand, E_s	=	197000 MPa [AASTHO LRFD `12, Art 5.4.4.2]
Area of each Strand	=	140 mm^2[TY Lin Page 56]
Area of each Cable	=	140 x 19 = 2660 mm^2
Jacking Force per Cable	=	1395 x 2660 = 3710 KN $\leq$ 0.9 f_y [1860 MPa*0.75 = 1395 MPa] [0.9*0.9-.06 = 0.75] [AASTHO LRFD `12, Table 5.9.3-1]

Number of Cable Initially Assumed = 3 Nos.

Calculation of Section Properties

Non-Composite Section

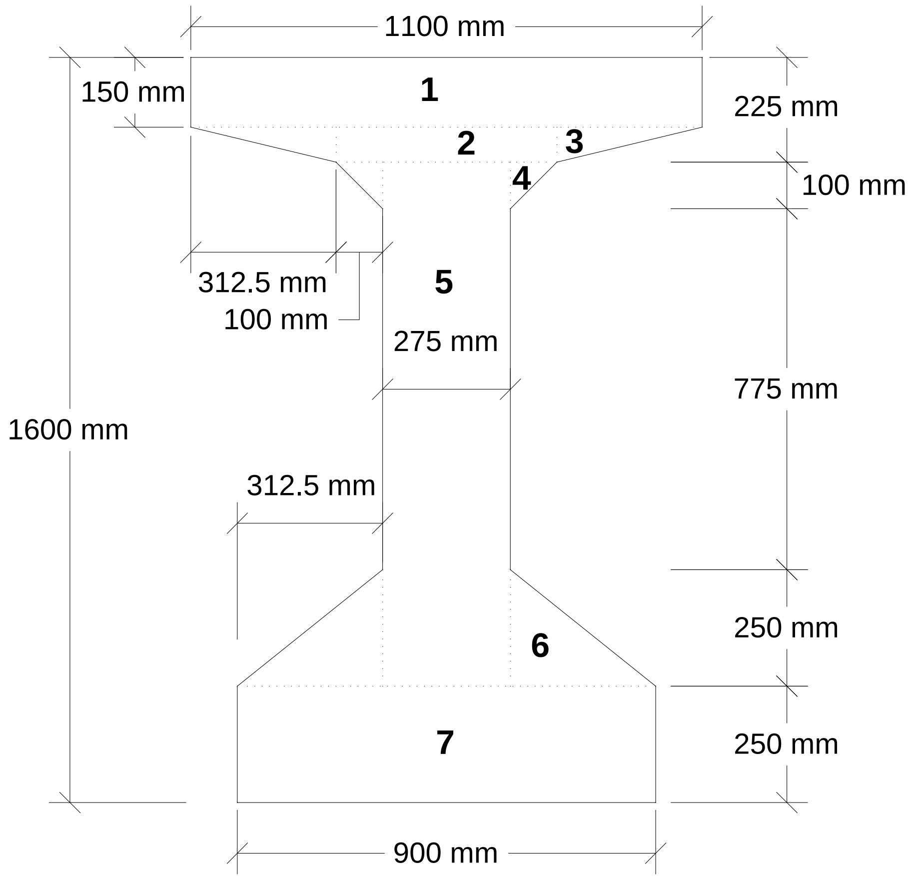


Figure 2: Non-Composite Section at Middle

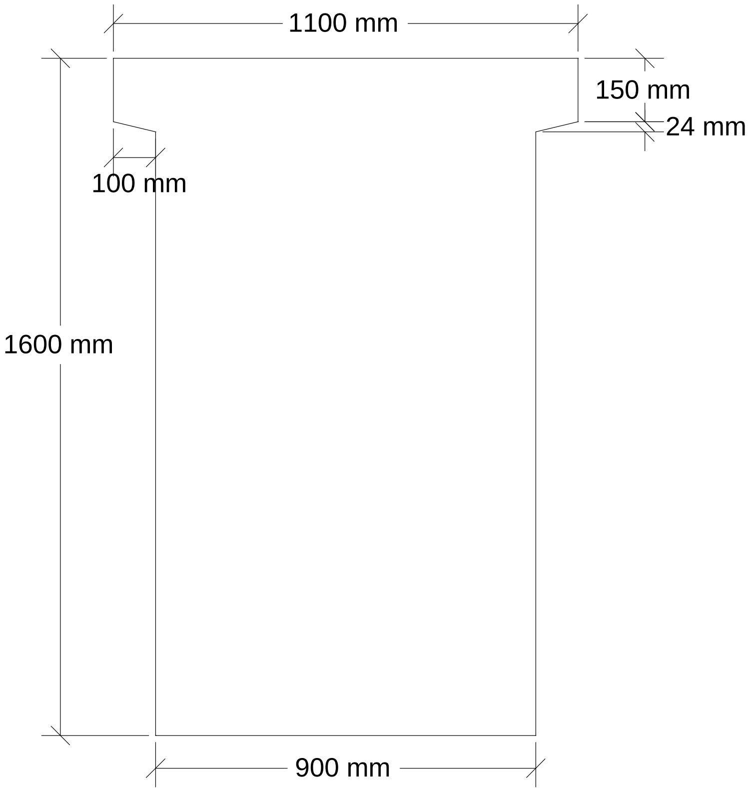


Figure 3: Non-Composite Section at End

Table 1: Section Property

Part	Shape	b	H	A	Y_{bot}	AY_{bot}	Y_b	I	Y	AY^2
Unit		mm	Mm	mm^2	mm	mm^3	mm	mm^4	mm	mm^4
1		1100	150	$1.65x10^5$	1525	$251.625x10^6$	763.75	$309.4x10^6$	761.25	$9.56x10^{10}$
2		475	75	35625	1412.5	$503.203x10^5$		$167x10^5$	648.75	$1.50x10^{10}$
3		312.5	75	23437.5	1425	$333.984x10^5$		$732.4x10^4$	661.25	$1.02x10^{10}$
4		100	100	10000	1341.7	$134.167x10^5$		$555.6x10^4$	577.95	$0.33x10^{10}$

5		275	1125	309375	787.5	243.633×10^{6}		326.3×10^{8}	23.75	$.017 \times 10^{10}$
6		312.5	250	78125	333.33	260.417×10^{5}		271.3×10^{6}	430.42	1.45×10^{10}
7		900	250	225000	125	281.25×10^{5}		117.2×10^{7}	638.75	9.18×10^{10}
Total				846562.5		646.560×10^{6}		344.1×10^{8}		23.1×10^{10}

Here,

Y_b = 763.75 mm = 0.764 m

Y_t = (1600 – 763.75) mm = 0.836 m

Area = 846562.5 mm^2 = 0.8466 m^2

MOI_{girder}, I_c = 2.654×10^{11} mm^4 = 0.2654 m^4

Section $Modulus_b$, Z_b = 0.3474 m^3 [$Z_b = I_c/Y_b$]

Section $Modulus_t$, Z_t = 0.3175 m^3 [$Z_t = I_c/Y_t$]

Kern $Point_t$, K_t = 0.41 m [$K_t = Z_b/A$]

Kern $Point_b$, K_b = 0.375 m [$K_b = Z_t/A$]

Composite Section

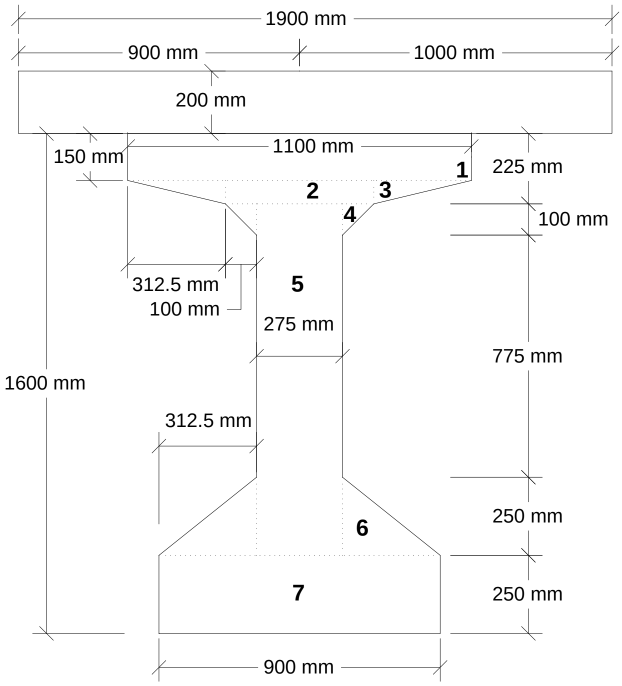


Figure 4: Composite Section at Middle

Modular Ratio, MOE of slab/girder	=	27690/31975 = 0.866
Effective Slab Width	=	1.9 m
Transformed Slab Width	=	0.866x1.9 = 1.6454 m
Transformed Slab Area	=	0.866x1.9x0.2 = 0.33 m^2

Table 2: Section Property

Part	A	Y_{bot}	AY_{bot}	Y'$_b$	I	Y	AY^2
	m^2	m	m^3	m	m^4	m	m^4
Girder	0.8466	0.764	0.6468	1.0265	0.2654	0.2625	0.0583
Slab	0.33	1.7	0.561		0.0011	0.6735	0.15
Total	1.1766		1.2078		0.2665		0.2083

Here,

Y'_b	=	1.0265 m
Y'_t	=	(1.6 – 1.0265) m = 0.5735 m
Y'_{ts}	=	(1.8 – 1.0265) m = 0.7735 m
Area	=	1.1766 m^2
MOI_{girder}, I'_c	=	0.4748 m^4
Section $Modulus_b$, Z'_b	=	0.4625 m^3 [$Z'_b=I'_c/Y'_b$]
Section $Modulus_t$, Z'_t	=	0.823 m^3 [$Z'_t=I'_c/Y'_t$]
Section $Modulus_{ts}$, Z'_{ts}	=	0.614 m^3[$Z'_{ts}=I'_c/Y'_{ts}$]
Kern $Point_t$, K'_t	=	0.393 m [$K'_t=Z'_b/A$]
Kern $Point_b$, K'_b	=	0.7 m [$K'_b=Z'_t/A$]
Constant $Factor_t$, m_t	=	0.386[$m_t=Z_t/Z'_t$]
Constant $Factor_b$, m_b	=	0.751[$m_b=Z_b/Z'_b$]

Concrete Volume in PC girder

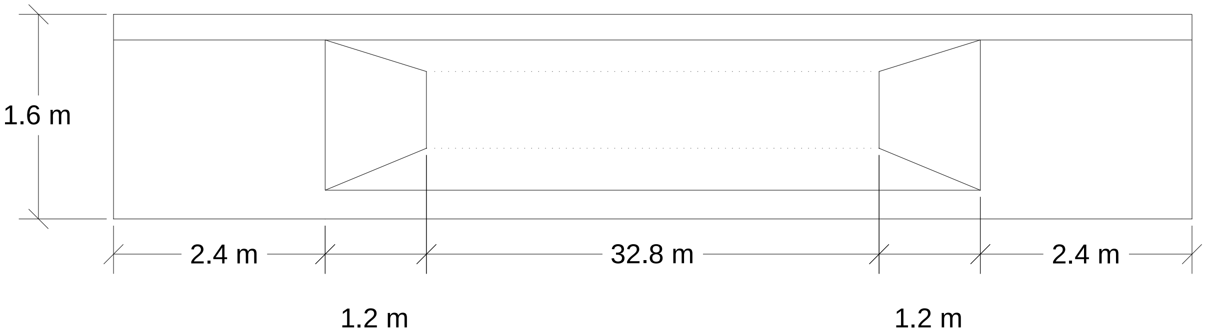


Figure 5: Elevation of PC Girder

Area of Mid-Block	=	0.8466 m^2
Area of End Block	=	1.47 m^2 [calculation not shown]
Area of Slopped Block	=	1.158 m^2 [Avg of Mid& End Block Area]
Total Volume of Girder	=	2.4x2x1.47+1.2x2x1.158+32.8x0.8466 m^3
	=	37.6 m^3

Moment & Shear Calculation

Calculation of Dead Load Moment

a) Dead Load Moment due to Girder

Load from Mid-Block = 0.8466x24 = 20.32 KN/m

Load from End Block = 1.47x24 = 35.28 KN/m

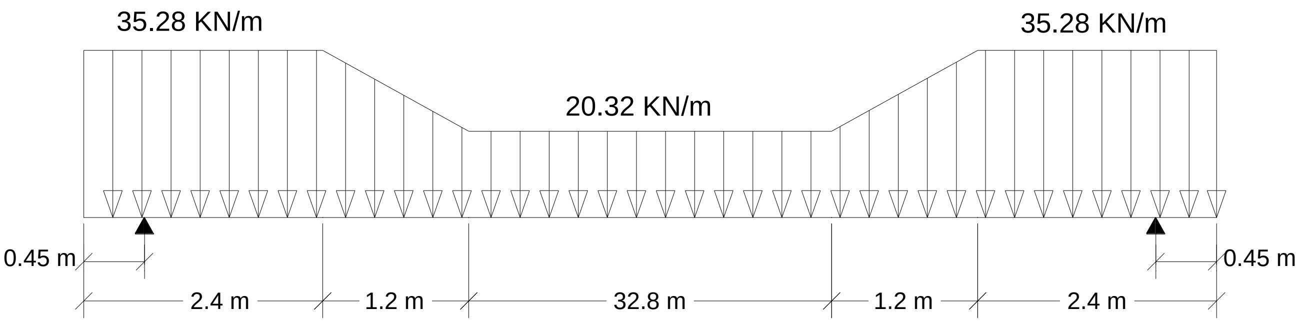


Figure 6: Self Weight of Girder

Reaction at Support = $35.28 \times 2.4 + \frac{35.28+20.32}{2} \times 1.2 + 20.32 \times \frac{32.8}{2} = 451.3$ KN

Moment at Mid Span, $M_{L/2} = 451.3 \times 19.55 - 35.28 \times 2.4 \times 18.8 - \frac{1.2\times(35.28-20.32)}{2} \times 17.2 - 20.32 \times 17.6 \times \frac{17.6}{2}$

$= 3929.5 \text{ KN.m}$

b) Dead Load Moment due to Cross Girder

Load from Exterior = 1.1 x (1-0.1375) x 0.5x24 = 11.4 KN

Load from Interior = 1.1 x (1-0.1375) x 0.3x24 = 6.83 KN

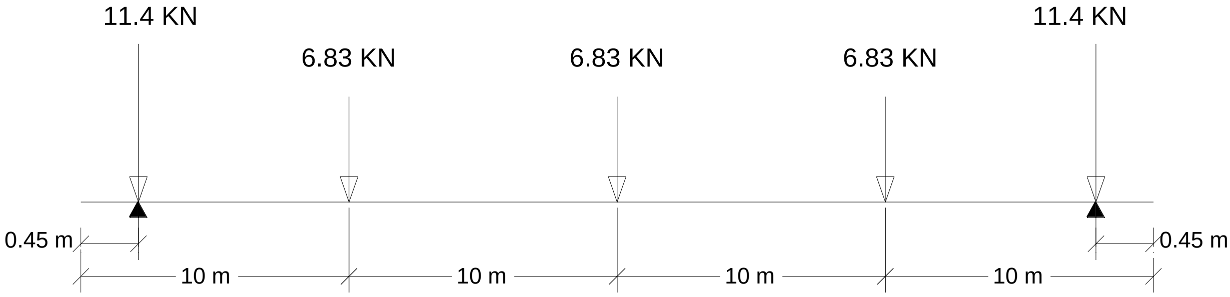


Figure 7: Load due to Cross Girder

Reaction at Support = $\frac{(2\times11.4+3\times6.83)}{2} = 21.645 \text{ KN}$

Moment at Mid Span, $M_{L/2} = 21.645 \times 19.55 - 11.4 \times 19.55 - 6.83 \times 10 = 132 \text{ KN.m}$

c) Dead Load Moment due to Deck Slab

Load from Deck, w = 1.9x0.2x24 = 9.12 KN/m

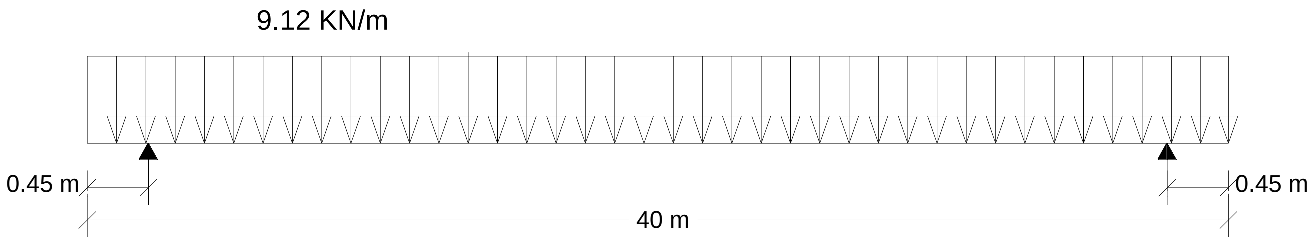


Figure 8: Self Weight of Deck Slab

Moment at Mid Span, $M_{L/2}$ = $wL^2/8 = (9.12\text{x}39.1^2)/8 = 1743$ KN-m

d) Dead Load Moment due to Other Loads

Load from Rail Bar = 1.08 KN/m [explanation needed]

Load from Rail Post = 0.672 KN/m [explanation needed]

Load from Wheel Guard, Post Base etc	=	1.35 KN/m	[explanation needed]
Load from Footpath, Walkway Slab etc	=	4.0 KN/m	[explanation needed]
Load from Wearing Course	=	0.748 KN/m	[0.650x0.05x23]
Sub Total	=	7.85 KN/m	

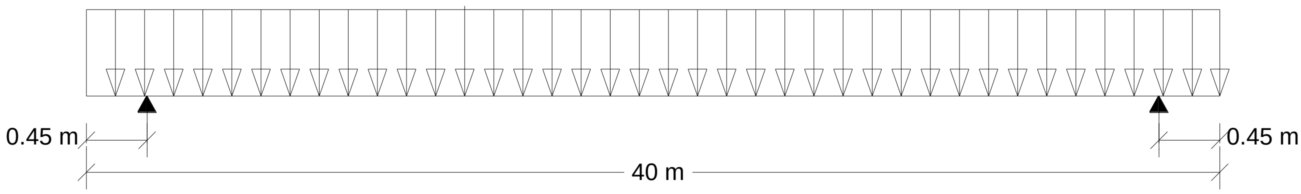


Figure 9: Other (Rail Bar, Rail Post, Wheel Guard, Post Base, Footpath, Walkway Slab and Wearing Course) Loads

Moment at Mid, $M_{L/2} = wL^2/8 = (7.85x39.1^2)/8 = 1500.1$ KN-m

e) Total Dead Load Moment

M_{DL} = Girder + Cross Girder + Deck Slab + Other

= 3929.5+132+1743+1500.1
= 7304.6 KN-m

f) Total Factored Dead Load Moment

MF_{DL} = (3929.5+132+1743) x 1.25 +1500.1x1.5
[AASTHO `12, Table 3.4.1-2]

= 9505.8 KN-m

Calculation of Live Load Moment

According to AASTHO LRFD HL 93 loading, each design lane should occupy either by the design truck or design tandem and lane load which will be effective 3000mm transversely within a design lane. [AASTHO `12 3.6.1.2.1]

Distribution Factor for Moment
For Interior Beam/ Girder

One Design Lane Loaded: $0.06+\left(\frac{S}{4300}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{Lt_s^3}\right)^{0.1}$ Two or More Design Lanes Loaded: $0.075+\left(\frac{S}{2900}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{Lt_s^3}\right)^{0.1}$	$1100 \le S \le 4900$ $110 \le t_s \le 300$ $6000 \le L \le 73\ 000$ $N_b \ge 4$ $4\times10^9 \le K_g \le 3\times10^{12}$
use lesser of the values obtained from the equation above with $N_b = 3$ or the lever rule	$N_b = 3$

[AASTHO `12, Table 4.6.2.2.2b-1]

Here,

S = Spacing of Main Girder
t_s = Thickness of Slab
L = Span Length
N_b = Number of Beam/ Girder
$K_g = n(I + Ae_g^2)$
$n = E_B/E_D$
E_B = Modulus of elasticity of beam/ girder material (MPa)
E_D = Modulus of elasticity of deck/ slab material (MPa)
I = Moment of inertia of beam/ girder (mm^4)
A= Area of Girder (mm^2)
e_g = distance between center of gravity of beam/ girder and deck/ slab (mm)

$$K_g = \frac{\sqrt{40}}{\sqrt{30}}\{2.654\times10^{11} + 846562.5\times(1800-763.75)^2\}$$
$$= 1.356\text{x}10^{12}$$

Distribution Factor for Moment, DFM $= 0.075 + (\frac{S}{2900})^{0.6}(\frac{S}{L})^{0.2}(\frac{k_g}{L\times{t_s}^3})^{0.1}$

$$=0.075 + (\frac{2000}{2900})^{0.6}(\frac{2000}{40000})^{0.2}(\frac{1.356\times10^{12}}{40000\times200^3})^{0.1}$$
$$= 0.583$$

For Exterior Beam/ Girder

One Design Lane Loaded	Two or More Design Lanes Loaded	Range of Applicability
Lever Rule	$g = e\, g_{interior}$ $e = 0.77 + \frac{d_e}{2800}$	$-300 \le d_e \le 1700$
	use lesser of the values obtained from the equation above with N_b = 3 or the lever rule	$N_b = 3$

[AASTHO `12, Table 4.6.2.2.2d-1]

Here,

g = distribution factor for exterior beam/ girder

g_{interior} = distribution factor for interior beam/ girder

e = eccentricity of a lane from center of gravity of the pattern of girders (mm)

d_e = distance from the exterior web of exterior beam to the interior edge of curb or traffic barrier (mm).

$$e = 0.77 + \frac{d_e}{2800} = 0.77 + \frac{4000 - \frac{7300}{2}}{2800} = 0.895 \text{ mm}$$

Distribution Factor for Moment, DFM, g = e x g_{interior} = 0.895 x 0.583 = 0.523

a) Moment due to Truck Load [AASTHO `12, Figure 3.6.1.2.2-1]

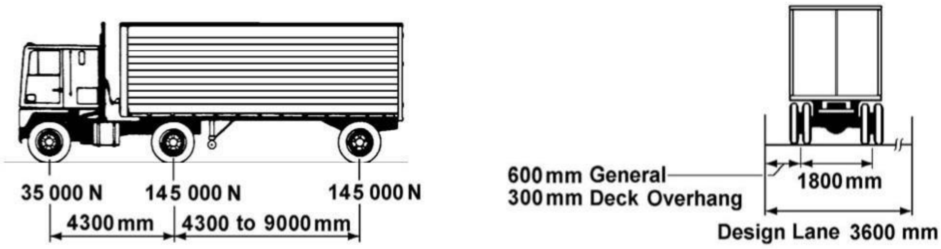


Figure 10: AASTHO HL-93 Truck Loading

Rear Axle Load = 145xDFM = 145x0.523 = 75.84 KN

Front Axle Load = 35xDFM = 35x0.523 = 18.31 KN

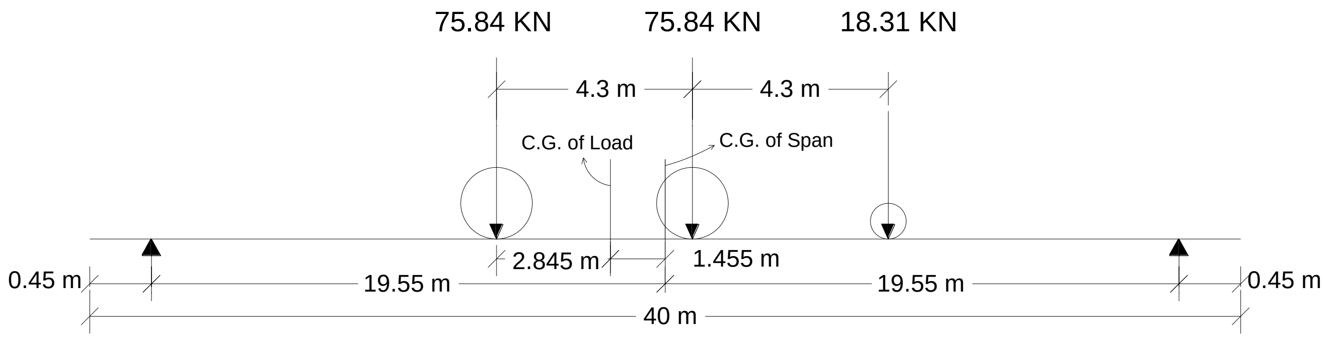


Figure 11: AASTHO HL-93 Truck Loading

$$\text{CG of Load } = \frac{75.84 \times 0 + 75.84 \times 4.3 + 18.31 \times 8.6}{75.84 \times 2 + 18.31} = 2.845 \text{ m}$$

$$\text{Reaction at Left Support } = \frac{75.84 \times 23.123 + 75.84 \times 18.823 + 18.31 \times 14.523}{39.1}$$

= 88.16 KN

Moment at Mid, $M_{L/2}$ = 88.16x19.55-75.84 x (4.3-0.727) = 1452.55 KN-m

Impact Moment = 1452.55 x 0.33 = 479.3 KN-m [AASTHO `12, Table 3.6.2.1-1]

Total Live Load Moment due to Truck Load = 1452.55+479.3 = 1931.85 KN-m **(Governs)**

b) Moment Due to Tandem Load [AASTHO `12, 3.6.1.2.3]

Wheel Load = 110x0.523 = 57.53 KN

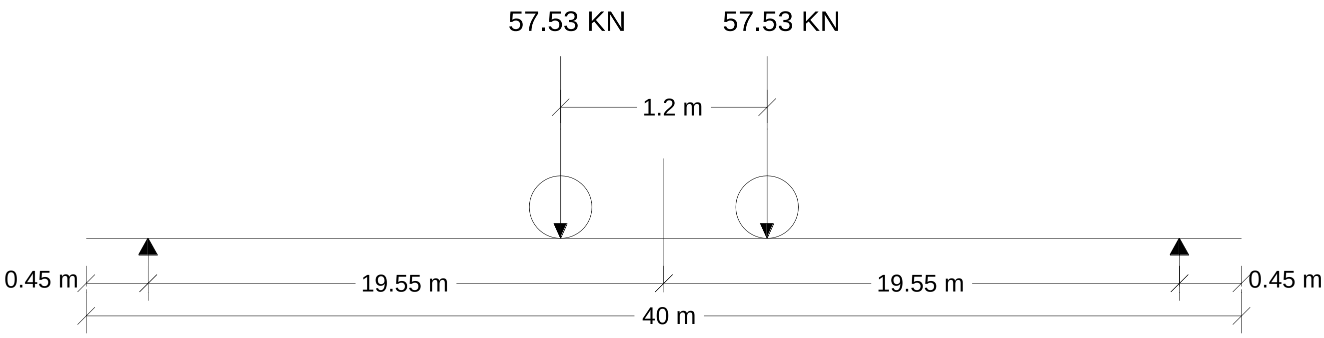


Figure 12: AASTHO Standard Tandem Loading

Moment at Mid, $M_{L/2}$ = 57.53x19.55-57.53x0.6 = 1090.2 KN-m

Impact Moment = 1090.2x0.33 = 359.8 KN-m

Total Live Load Moment due to Tandem Load = 1090.2+359.8 = 1450 KN-m **(Does not govern)**

c) Moment Due to Lane Load [AASTHO `12, 3.6.1.2.4]

Lane Load, w = 9.3xDFM = 9.3x.523 = 4.864 KN/m

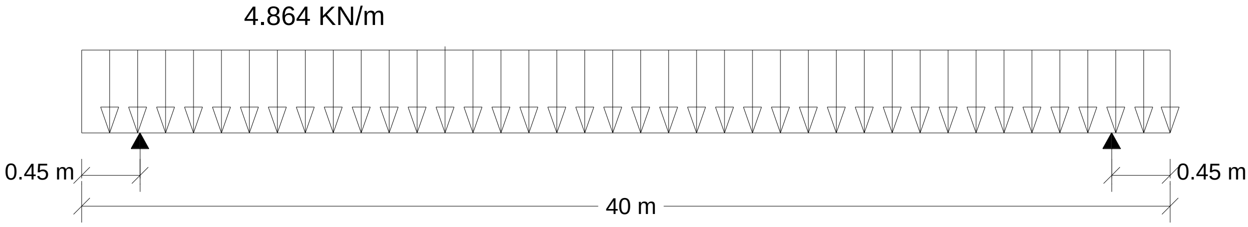


Figure 13: Lane Load

Moment at Mid, $M_{L/2}$ = $wL^2/8$ = $(4.864x39.1^2)/8$ = 929.5 KN-m

d) Total Live Load Moment

As Truck Load Moment is higher than Tandem Load Moment, the total vehicular live load moment as stated in AASTHO,

M_{LL} = Truck Load + Lane Load

= 1931.85+929.5 = 2861.35 KN-m

e) Factored Total Live Load Moment

M_{FLL} = 2861.35 x 1.75 = 5007.36 KN-m [AASTHO `12, Table 3.4.1-1]

Shear Calculation

a) Shear due to Dead Load

Shear due to Self-Weight of Girder = 451.3 KN

Shear due to Cross Girder = 21.645 KN

Shear due to Deck Slab = 9.12x (39.1/2) KN = 178.3 KN

Shear due to Other Loads = 7.85x (39.1/2) KN = 153.47 KN

Total Dead Load Shear = (451.3+21.645+178.3+153.47) KN

= 804.72 KN

b) Distribution Factor for Shear [AASTHO `12, Table 4.6.2.2.3a-1]

For Interior Beam/ Girder

One Design Lane Loaded	Two or More Design Lanes Loaded	Range of Applicability
$0.36+\frac{S}{7600}$	$0.2+\frac{S}{3600}-\left(\frac{S}{10\,700}\right)^{2.0}$	$1100 \le S \le 4900$ $6000 \le L \le 73\,000$ $110 \le t_s \le 300$ $N_b \ge 4$
Lever Rule	Lever Rule	$N_b = 3$

[AASTHO `12, Table 4.6.2.2.3a-1]

Distribution Factor for Shear, DFV $= 0.2 + \frac{2000}{3600} - \left(\frac{2000}{10700}\right)^2 = 0.721$

For Exterior Beam/ Girder

One Design Lane Loaded	Two or More Design Lanes Loaded	Range of Applicability
Lever Rule	$g = e\, g_{interior}$ $e = 0.6 + \frac{d_e}{3000}$	$-300 \le d_e \le 1700$
	Lever Rule	$N_b = 3$

[AASTHO `12, Table 4.6.2.2.3b-1]

$$e = 0.6 + \frac{d_e}{3000} = 0.6 + \frac{4000 - \frac{7300}{2}}{3000} = 0.717$$

Distribution Factor for Shear, $DFV, g = e \times g_{interior}$= 0.717x0.721 = 0.517

c) Shear due to Live Load

i. Shear due to Truck Load

Rear Axle Load = 145 x DFV = 145 x 0.517 = 75 KN

Front Axle Load = 35 x DFV = 35 x 0.517 = 18.1 KN

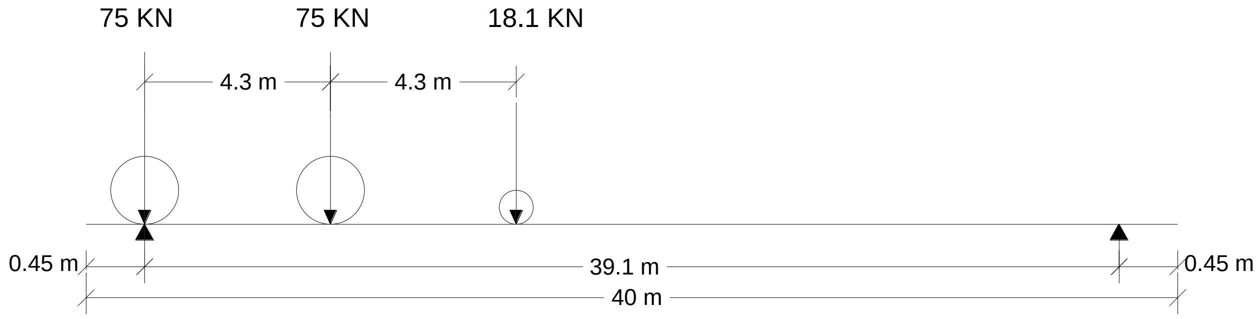


Figure 14: AASTHO HL-93 Truck Loading

Reaction at Left Support = $\frac{75\times39.1+75\times34.8+18.1\times30.5}{39.1} = 155.87$ KN

Impact Shear = (155.87x0.33) = 51.44 KN

Total Shear due to Truck = (155.87+51.44) KN = 207.31 KN**(Governs)**

ii. Shear due to Tandem Load

Wheel Load = 110x0.517 = 56.87 KN

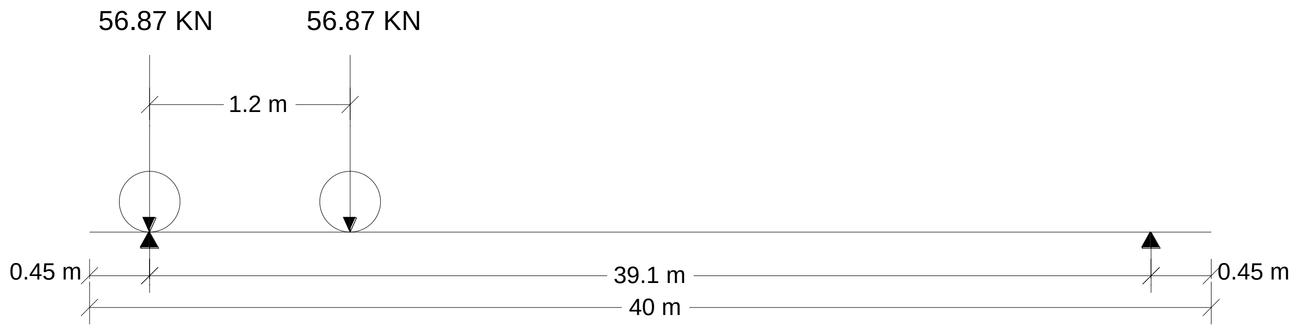


Figure 15: AASTHO Standard Tandem Loading

Reaction at Left Support = $\frac{56.87\times39.1+56.87\times37.9}{39.1} = 112$ KN

Impact Shear = (112x0.33) = 36.96 KN

Total Shear due to Tandem = (112+36.96) KN = 148.96 KN **(Does not govern)**

iii. Shear due to Lane Load

Load due to lane per unit Length = 9.3x0.517 KN/m =4.81 KN/m

Shear Due to Lane Load = (4.81x39.1)/2 KN = 94.04 KN

Total Live Load Shear = (207.31+94.04) KN = 301.35 KN

d) Total Shear

V_{D+L} = 804.72+301.35 = 1106.07 KN

e) Total Factored Shear

$V_{F(D+L)}$ = (804.72x1.5+301.35x1.75) = 1734.44 KN [Ref]

Estimation of Required Pre-Stressed Force and Number of Cable

Assumed Number of Cable = 3

CG of the Cable at Girder Mid = $\frac{2660\times3\times135}{2660\times3} = 135 \text{ mm} = 0.135 \text{ m}$

Eccentricity at Mid-Section = (0.764-0.135) m = 0.629 m

$$\text{Required Prestress Force, F} = \frac{M_p + M_C \times m_b - f'_b \times k_t \times A_c}{e + k_t}$$

For full pre-stressing, $f'_b = 0$

[Design of Prestressed Concrete Structures, T.Y. Lin, Chapter 6, Equation 6-18]

Here,

M_P = Moment due to Girder, Cross Girder & Deck Slab [Precast]

M_C = Moment due to Live Load & Wearing Course [Composite, Service Condition]

$$= \frac{(3929.5 + 132 + 1743) + (2810.1 + 143) \times 0.751}{0.629 + 0.41}$$

= 7721.2 KN

$$\text{Required Steel Area, } A_s = \frac{7721.2 \times 1000}{0.6 * 1860} = 6918.6 \text{ mm}^2$$

Using 19K15 Strand, Required Cable = $(\frac{6918.6}{2660})$

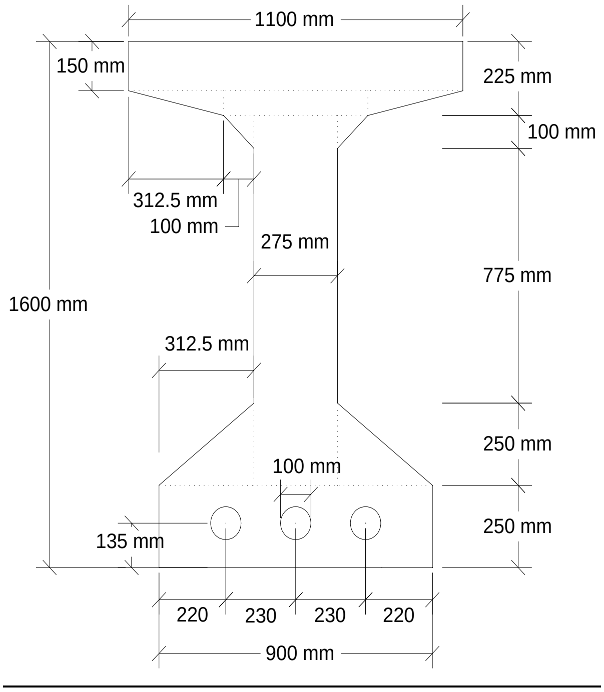
= 2.6 Nos.

Here, Pre-stressing is done in single stage

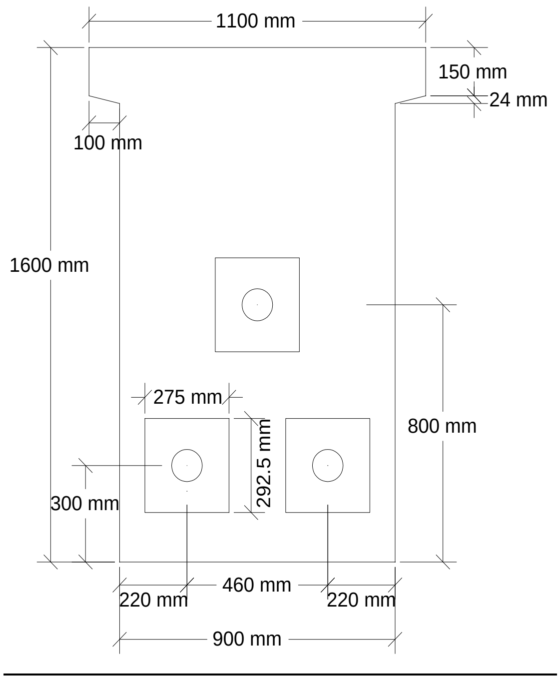
Jacking Force = 3710x3 = 11130 KN

Eccentricity of Girder Section = (0.764-0.135) m = 0.629 m

Eccentricity of Composite Section = (1.0265-0.135) m = 0.8915 m



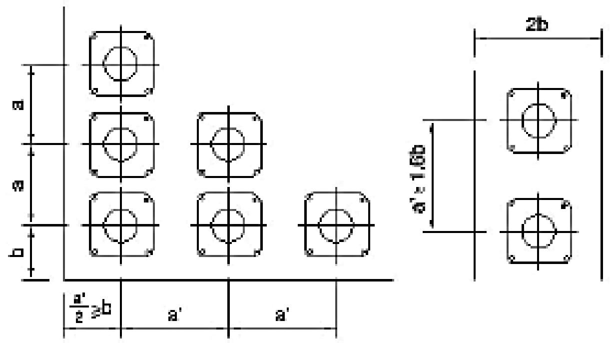
a) Middle Section



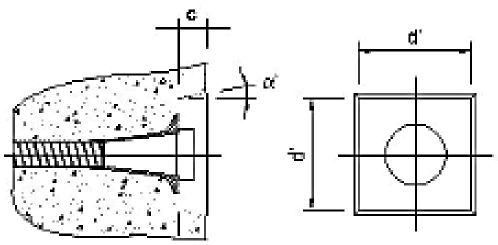
b) End Section

Figure 16: Cable arrangement in Mid and End Section of Girder

ANCHORAGE SPACING & EDGE DISTANCE



ANCHORAGE CENTERS & EDGE CLEARANCES



RECESS DIMENSIONS

CONC. CUBE STRENGTH in N/mm.²

ANCHORAGE TYPE	TENDON FORCE kN	BASE SQUARE OF GUIDE mm. X mm.	M30		M35		M40		M45		M50		M55		M60		RECESS DIMENSIONS			
			a	b	a	b	a	b	a	b	a	b	a	b	a	b	c	d'	Jack	α°
4K13	734.8	147 X 147	200	150	185	135	170	120	170	120	170	120	170	120	170	120	100	205	K100	10
4K15	1042.8	160 X 160	220	170	200	150	180	130	180	130	180	130	180	130	180	130	110	220/	K100	10
7K13	1285.9		220	170	200	150	180	130	180	130	180	130	180	130	180	130	110	270	K200	10
7K15	1824.9	220 X 220	280	220	260	200	240	180	240	180	240	180	240	180	240	180	120	270/	K200	10
12K13	2204.4		280	220	260	200	240	180	240	180	240	180	240	180	240	180	120	300	K350	10
12K15	3128.4	244 X 244	325	250	300	230	290	210	270	185	270	185	270	185	270	185	125	300/	K350	10
19K13	3490.3		325	250	300	230	290	210	270	185	270	185	270	185	270	185	125	350	K500	10
19K15	4953.3	275 X 292.5	430	320	400	280	360	250	320	220	320	220	320	220	320	220	125	350/	K500	10
27K13	4959.9		430	320	400	280	360	250	320	220	320	220	320	220	320	220	125	500	K700	10
37K13	6796.9	365 X 365	520	400	490	360	460	330	430	300	400	270	400	270	400	270	145	500	K700	30
27K15	7038.9		520	400	490	360	460	330	430	300	400	270	400	270	400	270	145	500	K700	30
37K15	9645.9	425 X 425	580	440	560	400	540	380	500	350	460	320	460	320	460	320	180	600	K1000	30

Assumptions:

a. The minimum distances between anchorages "a" and the minimum distance "b" from the nearest free edge are given for various concrete strength levels.

b. The minimum distance "a" must be combined with a distance between a' > a in the orthogonal direction to provide for each anchorage a rectangular distribution area that satisfies the conditions $aa' \geq 3b^2$.

c. **M30 / M35 / M40 /....** are Characteristic Breaking Strength of Concrete Cube after 28 days.

d. Nominal Breaking Strength: for Ø 15.2mm. Strand – 260.7 kN.
for Ø 12.7mm. Strand – 183.7 kN.

e. Recess dimension d' (eg.220/270) are shown with basic Jack/Optional Jack.

f. All Dimensions shown here are in millimeters.

[Ref-Prestressing Manual 'THE FREYSSINET PRESTRESSED CONCRETE CO. LTD'.]

Calculation of Loss

Instantaneous Loss

a) Friction and Anchorage Slip Loss

Table 3: Friction and Wedge Pull Loss

Cable No	Vertical Sag, d_r	Horizontal Sag	Radius of Curvature, R	α=X/R (X=1m)	α=X/R (X=L/2)
1	165	0	1158.2	8.63×10^{-4}	0.01688
2	165	0	1158.2	8.63×10^{-4}	0.01688
3	665	0	287.4	3.48×10^{-3}	0.06802

Cable No	Initial Pre-stress Force, KN	Loss (per m), KN	Loss (L/2), KN	Wedge Pull Effect Distance, X_A	Loss due to Wedge Pull (if $X_A > L/2$	Friction + Wedge Pull Loss (%)
1	3710	2.61	63	34.71	79.14	3.83
2		2.61	63	34.71	79.14	3.83
3		24.4	109.3	11.35	0	2.95
				Percent Loss =		3.54(Average)

Friction Loss [Sample Calculation of Cable 1]

Friction Loss, $\Delta f_{pF} = \Delta f_{pi}\,[1 - e^{-(kx+\mu\alpha)}]$[AASTHO `12, Equation 5.9.5.2.2b-1]

Here,

Friction Co-Efficient, μ = 0.25 [AASTHO `12, Table 5.9.5.2.2b-1]

Wobble Co-Efficient, k = 0.00066 /m

$\alpha = \sqrt{\alpha_v{}^2 + \alpha_H{}^2} \qquad \alpha = \frac{X}{R}$

$$\text{Radius of Curvature, R} = \frac{L^2}{8 \times d_r} = \frac{39.1^2}{8 \times 0.165} = 1158.2 \text{ rad}$$

$$d_r = \text{vertical sag height} = 300 - 135 = 165 \text{ mm} = 0.165 \text{ m}$$

$$\alpha = \frac{X}{R} = \frac{1}{1158.2} = 8.63 \times 10^{-4}; \quad X = 1 \text{ m}$$

$$\alpha = \frac{X}{R} = \frac{19.55}{1158.2} = 0.0169; \quad X = \frac{L}{2} = 19.55 \text{ m}$$

$$\text{Friction Loss}, \Delta f_{pF} = 3710\left[1 - e^{-\left(0.00066\times1+0.25\times8.63\times10^{-4}\right)}\right] = 2.61\ \text{KN};\ X = 1\ \text{m}$$

$$\text{Friction Loss}, \Delta f_{pF} = 3710\left[1 - e^{-\left(0.00066\times19.55+0.25\times0.0169\right)}\right] = 63\ \text{KN};\ X = 19.55\ \text{m}$$

Anchorage Slip Loss [Calculation of Cable 1]

Let's Assume, Anchorage Slip = 6 mm [AASHTO `12, C5.9.5.2.1]

Distance till which anchorage slip loss will be effective,

$$X_A = \sqrt{\frac{\text{Slip} \times E_p \times A_s}{\Delta f_{pF}}} = \sqrt{\frac{0.006 \times 197 \times 2660}{2.61}} = 34.71\ \text{m}$$

Anchorage Slip loss will only occur when $X_A > \frac{L}{2}$

$$\text{Loss due to slip} = 2 \times \Delta f_{pF} \times \left(X_A - \frac{L}{2}\right) = 2 \times 2.61 \times (34.71 - 19.55) = 79.14\ \text{KN}$$

Friction and Slip Loss = 63 + 79.14 = 142.14 KN

$$\text{Percentage (\%)} = \frac{142.14}{3710} \times 100 = 3.83\%$$

b) Elastic Shortening Loss [AASTHO `12, 5.9.5.2.3b-1]

$$\Delta f_{PES} = \frac{N-1}{2N} \frac{E_p}{E_{ci}} f_{cgp}$$

Here,

N=Number of identical pre-stressing tendons;

f_{cgp}=Sum of concrete stresses at the center of gravity of pre-stressing tendons due to the pre-stressing force after jacking and the self-weight of the member at the sections of the maximum moments (MPa)

$$f_{cgp} = \frac{P_{eff}}{A} + \frac{P_{eff} \times e^2}{I} - \frac{M_g \times e}{I}$$

$$= \frac{3579 \times 3}{0.8466} + \frac{3579 \times 3 \times 0.629^2}{0.2654} - \frac{3929.5 \times 0.629}{0.2654} = 19375.6\frac{\text{KN}}{\text{m}^2} = 19.376\ \text{MPa}$$

[3710 KN x (100% - 3.54%) = 3579 KN]

$$\Delta f_{PES} = \frac{3-1}{2 \times 3} \times \frac{197 \times 10^3}{30335} \times 19.376 = 41.94\ \text{MPa}$$

$$\text{Percent of Elastic Shortening Loss} = \frac{41.94 \times 2660}{3710 \times 1000} \times 100 = 3\ \%$$

$$\text{Total Instantaneous Loss} = 3.54 + 3 = 6.54\ \%$$

Long Term / Time Dependent Loss

Approximate Estimate of Time-Dependent Losses, [AASTHO`12, 5.9.5.3-1]

Long Term Loss due to Shrinkage, Creep and Still Relaxation is given below.

$$\Delta f_{pLT} = 10 \times \frac{f_{pi} \times A_{ps}}{A_g} \times \gamma_h \gamma_{st} + 83 \gamma_h \gamma_{st} + \Delta f_{pR}$$

Where,

$\gamma_h = 1.7 - 0.01H,$ Relative Humidity, $H = 70\%$

$$\gamma_{st} = \frac{35}{7 + f'_{ci}} = \frac{35}{7 + 36} = 0.814$$

Here,

f_{pi} = prestressing steel stress immediately prior to transfer (MPa)

H = the average annual ambient relative humidity (%)

γ_h = correction factor for relative humidity of the ambient air

γ_{st} = correction factor for specified concrete strength at time of prestress transfer to the concrete member

Δf_{pR} = an estimate of relaxation loss taken as 17 MPa for low relaxation strand, 70 MPa for stress relieved strand, and in accordance with manufacturers recommendation for other types of strand (MPa)

$$\Delta f_{pLT} = \left[\frac{10 \times 0.75 \times 1860 \times 2660}{1.176 \times 10^6} \times (1.7 - 0.01 \times 70\%) \times \left(\frac{35}{7 + 36}\right) + 83 \times (1.7 - 0.01 \times 70\%) \times \left(\frac{35}{7 + 36}\right) + 17\right] = [25.68 + 67.56 + 17] = 110.24 \text{ MPa}$$

$$\text{Percent of Time Dependent Loss} = \frac{110.24 \times 2660}{3710 \times 1000} * 100 = 7.9\%$$

Total Loss (Instantaneous & Time Dependent) = (6.54+7.9) % = 14.44%

Revised No of Required Cable

Total Percent of Loss = 14.44%

Total Loss = 1395*14.44% MPa= 201.438 Mpa

Effective Steel Stress after Loss = 0.75*1860-201.438 =1193.562 MPa

Revised No of Required Cable,

$$\frac{\text{Requred Effective Force} * 1000}{\text{A}_{\text{ps}} * \text{Effective Steel Stress after Loss}} = \frac{7721.2 * 1000}{2660 * 1193.562} = 2.43 \text{ Nos.}$$

Actual Effective Force per Cable = 1193.562x (2660/1000) =3174.87 KN

Actual Effective Stress per Cable = 1193.562 MPa

Stress Calculation

Table 4: Calculation of Stress in different stages

Calculation of Stress	
Stress due to Self-Weight of Girder [+ = Compressive - = Tension]	$\sigma = \left(\frac{M_g * Y}{I_g}\right) * \frac{1}{1000}$ $\sigma_b = -\frac{3929.5 * .764}{.2654 * 1000} = -11.31\ \text{MPa}$ $\sigma_t = +\frac{3929 * .837}{.2654 * 1000} = +12.39\ \text{MPa}$
Stress due to PS Force	$\sigma = \left(+\frac{\text{Jacking Force}}{A_g} \pm \frac{\text{Jacking Force} * e}{Z_b}\right) * \frac{1}{1000}$ $\sigma_b = \left(+\frac{3710 * 3}{.8466} + \frac{3710 * .629 * 3}{.3474}\right) = 33.298\ \text{MPa}$ $\sigma_t = \left(+\frac{3710 * 3}{.8466} - \frac{3710 * .629 * 3}{.3175}\right) = -8.9029\text{MPa}$
Stress due to Friction and Slip loss (3.54%)	$\sigma_b = -33.298 * 3.54\% = -1.17\ \text{MPa}$ $\sigma_t = -(-8.09029) * 3.54\% = +0.315\ \text{MPa}$
Stress due to Elastic Shortening (3%)	$\sigma_b = -33.298 * 3\% = -.998\text{MPa}$ $\sigma_t = -(-8.903) * 3\% = +0.267\ \text{MPa}$
Stress due to $\frac{1}{3}$ Time Dependent Loss of PS (7.9/3%)	$\sigma_b = -33.298 * 2.63\% = -0.876\ \text{MPa}$ $\sigma_t = -(-8.903) * 2.63\% = +0.234\ \text{MPa}$
Stress due to Self-Weight of Deck Slab	$\sigma_b = -\frac{1743 * .764}{.2654 * 1000} = -5.0175\ \text{MPa}$ $\sigma_t = +\frac{1743 * .836}{.2654 * 1000} = +5.49\ \text{MPa}$

Stress due to Self-Weight of Cross Girder	$\sigma_b = -\frac{132 * .764}{.2654 * 1000} = -.3799$ MPa $\sigma_t = +\frac{132 * .836}{.2654 * 1000} = +.4157$ MPa
Stress due to Other $\frac{2}{3}$ Time (Composite) Dependent Loss of PS (7.9X 2/3%)	$\sigma_b = -\left(\frac{3710 * 3 * 5.26\%}{1.1766}\right) - \left(\frac{3710 * 3 * 5.26\% * .8915}{.4625}\right) = -1.626$ MPa $\sigma_t = -\left(\frac{3710 * 3 * 5.26\%}{1.1766}\right) + \left(\frac{3710 * 3 * 5.26\% * .8915}{.823}\right) = +0.1366$ MPa $\sigma_{st} = -\left(\frac{3710 * 3 * 5.26\%}{1.1766}\right) + \left(\frac{3710 * 3 * 5.26\% * .8915}{.614}\right) = +0.3525$ MPa
Stress Due to Differential Shrinkage of Deck Slab	Tensile Stress in-situ Slab, $T = 1 * 10^{-4} * E_c \sqrt{\frac{f_s}{f_g}}$ So, $T = 1 * 10^{-4} * 31975 * \sqrt{\frac{30}{40}} = 2.769$ MPa Compressive Force at CG of Slab, P = T x S x t_s x 1000 = 2.769 x 1.9 x 0.2 x 1000 = 1052.22 KN CG of Slab from Composite Y_t =, (Y'_t- t_s/2) = (0.7735-0.2/2) = .6735 m $\sigma_b = \frac{P}{A_c} - \frac{P*.6735}{Z_b'} = \frac{1052.22}{1.1766} - \frac{1052.22 * .6735}{.4625} = -0.638$ MPa $\sigma_t = \frac{P}{A_c} + \frac{P*.6735}{Z_t'} = \frac{1052.22}{1.1766} + \frac{1052.22 * .6735}{.823} = +1.755$ MPa σ_{st} = (Stress Girder Top Fiber - Tensile Stress in-situ Slab T) = +1.755-2.769 = - 1.014 MPa

Stress due to Self-Weight of Wearing Course	$\sigma_b = -\frac{143 * 1.0265}{.4748 * 1000} = -.309$ MPa $\sigma_t = +\frac{143 * .5735}{.4748 * 1000} = +0.1727$ MPa $\sigma_{ts} = \frac{143}{.614 * 1000} = +.2328$ MPa
Stress due to Design Live Load	$\sigma_b = -\frac{2861.35 * 1.0265}{.4748 * 1000} = -6.186$ MPa $\sigma_t = \frac{2861.35 * .5735}{.4748 * 1000} = +3.456$ MPa $\sigma_{st} = \frac{2861.35}{.614 * 1000} = +4.66$ MPa

Table 5: Schedule of Stress

	Case	Stage of Stress	F_{bottom} (MPa)	F_{top} (MPa)	$F_{slab\ top}$ (MPa)
R1	Effect due to Self-Weight of Girder and PS transfer after IL	Dead Load of Naked Girder	-11.31	+12.39	0
		PS transfer	+33.298	-8.9029	0
		Instantaneous Loss (Friction+Slip+ES)	-2.168	+0.582	0
		Resultant of PS	**+19.82**	**+4.069**	**0**
		Permissible of PS	$+.6*f_{ci}' = .6*36 = +21.6$	$-0.25*(f_{ci}')^{1/2} = .25*36^{.5} = -1.5$	0
Comment			Okay	Okay	-
R2	Effect due to ½ of TDL	1/3 Time Dependent Loss of PS	-0.876	+0.234	0
		Resultant after ½ TDL of PS-I	**+18.944**	**+4.303**	0
		Permissible of PS-II	$+.6*f_c' = .6*40 = +24$	$-0.25*(fc')^{1/2} = -.25*40^{.5} = -1.58$	0
Comment			Okay	Okay	-
R3	Effect due to self-	Dead Load of Deck Slab	-5.0175	+5.49	0

		Dead Load of Cross Girder	-.3799	+.4157	0
		Resultant Stress	**+13.5466**	**+10.2087**	0
R4	Effect due to other 2/3 TDL of PS	2/3 Time Dependent Loss of PS (Composite)	-1.626	+0.1366	+0.3525
		Resultant Stress	**+11.9206**	**+10.3453**	**+0.3525**
R5	Effect due to Self-Weight of Wearing Course and Stress due to DS	Stress due to Differential Shrinkage	-0.638	+1.755	- 1.014
		Wearing Course on Composite Section	-0.309	+0.1727	+.2328
		Resultant on Composite	**+10.9736**	**+12.273**	**-0.4287**
Permissible Stress		**F_{bottom} (MPa)**	**F_{top} (MPa)**	**$F_{slab\ top}$ (MPa)**	**$F_{slab\ bottom}$ (MPa)**
		$+.5*f_c' = .5*40 = +20$	$-0.5*(fc')^{1/2} = -.5*40^{.5} = -3.16$	$+.45*f_{c(Slab)}' = .45*30 = +13.5$	$-0.5*(fc')^{1/2} = .45*30 = -2.73$
Comment		Okay	Okay	Okay	Okay

R6	Effect due to Live Load for Service	Stress due to Design Live Load	-6.186	+3.456	+4.66
		Resultant Stress, Service (Total DL+PS+Live Load)	**+4.7876**	**+15.729**	**+4.2313**
Permissible Stress at Service		**F_{bottom} (MPa)**	**F_{top} (MPa)**	**$F_{slab\ top}$ (MPa)**	**$F_{slab\ bottom}$ (MPa)**
		$+.6*f_c' = .6*40 = +24$	$-0.5*(fc')^{1/2} = -.5*40^{.5} = -3.16$	$+.6*f_{c(slab)}' = .6*30 = +18$	$-0.5*(fc')^{1/2} = .45*30 = -2.73$
Comment		Okay	Okay	Okay	Okay

Checking of Moment Capacity

Factored Moment (DL+LL) = (13259.07+61429.22) = 74688.28 KN-m

$$k = 2\left(1.04 - \frac{f_{pu}}{f_{py}}\right) = 2(1.04 - 0.9) = 0.28$$

$$d_p = 3500 + 200 - 503.33 = 3196mm$$

Let's Assume Rectangular Behavior,

$$c = \frac{(A_{ps}f_{pu} + A_s f_y - A_s' f_y'}{\left(0.85f'_c\beta_1 b + \frac{kA_{ps}f_{pu}}{d_p}\right)}$$

$$= \frac{2660 * 9 * 1860}{\left(0.85 * 40 * 0.76 * 1900 + \frac{0.28 * 2660 * 1860 * 9}{3196}\right)} = 840 > 200, T\ action$$

Let's Assume T behavior,

$$c = \frac{(A_{ps}f_{pu} + A_s f_s - A_s' f'_s - 0.85f'_c(b - b_w)h_f}{\left(0.85f'_c\beta_1 b_w + \frac{kA_{ps}f_{pu}}{d_p}\right)}$$

$$= \frac{2660 * 9 * 1860 - 0.85 * 45\ (1900 - 400) * 200}{\left(0.85 * 40 * 0.76 * 400 + \frac{0.28 * 2660 * 1860 * 9}{3196}\right)} = 2.32m$$

$$\alpha = \beta_1 * c = 0.76 * 2.35 = 1.76\ m$$

$$f_{ps} = f_{pu}\left(1 - \frac{k * c}{d_p}\right) = 1860\left(1 - \frac{0.28 * 2.32}{3.196}\right) = 1480\ MPa$$

$$M_n = A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_s\left(d_s - \frac{a}{2}\right) + A_s' f_y'\left(d'_s - \frac{a}{2}\right) + 0.85f'_c(b - b_w)h_f\left(\frac{a}{2} - \frac{h_f}{2}\right)$$

$$M_n = 2660 * 9 * 1480 * \left(3196 - \frac{1760}{2}\right) + 0.85 * 40 * (1900 - 400) * 200 * \left(\frac{1760}{2} - \frac{200}{2}\right)$$
$$= 90015\ KN - m$$

$$M_r = \emptyset * M_n = 0.9 * 90015 = 81015\ KN - m$$

$$M_u = 74688\ KN - m$$

$$M_r > M_u\ (ok)$$

Deflection Calculation

CG of cable (7 Nos.) at PS-I = 610 mm

CG of cable (2 Nos.) at PS-II = 130 mm

CG of Cable at Girder End at PS-I = 1925 mm

CG of Cable at Girder End at PS-II = 225 mm

a) Deflection due to self-weight of girder

$$\text{Self} - \text{weight w } = \frac{\text{volumn} * \text{ unit weight}}{\text{span}} = \frac{150.2675 \text{ x}24}{73} = 49.40 \text{ KN/m}$$

$$\text{Deflection} = \frac{5\text{wL}^4}{384 \text{ x Eci x Ic}} = \frac{5 \text{ x } 49.40 \text{ x}73^4}{384 \text{ x } 26290 \text{ x } 2.59} = 268 \text{ mm}$$

b) Deflection due to PS-I

CG of Cable at Girder End = 1925 mm

Sag Height = (1925 – 610) mm = 1315 mm

Eccentricity of End Section = (1797-1925) mm = -118 mm

$$\text{Average Prestressed Force after IL} = 3710 \text{ x } 7\left(1 - \frac{5.23}{100} - \frac{3.71}{100}\right) = 23648 \text{ KN}$$

$$\text{Equivalent upward UDL due to Cable Parabola} = \frac{8 \text{ x p x sag}}{\text{L}^2} = \frac{8 \text{ x } 23648 \text{ x } 1.315}{73^2}$$
$$= 46.68 \text{ KN/m}$$

Upward deflection due to PS-I transfer,

$$\frac{5\text{wL}^4}{384 \text{ x Eci x I}} = \frac{5 \text{ x } 46.68 \text{ x } 73^4}{384 \text{ x } 26290 \text{ x } 2.59} = 253.5 \text{ mm}$$

Deflection at Girder End eccentricity,

$$\frac{\text{p x e}_{\text{end x}}\text{L}^2}{8 \text{ x Eci x I}} = \frac{23648 \text{ x } (-0.118) \text{ x } 73^2}{8 \text{ x } 26290 \text{ x } 2.59} = -27.3 \text{ mm}$$

c) Deflection due to PS-II

CG of Cable in PS-II at girder end = 225 mm

Sag Height = (225 – 130) = 95 mm

Eccentricity at PS-II cable at girder end = (1.797-0.225) =1.572 m

$$\text{Average Prestressed Force after IL} = 3710 \text{ x } 2\left(1 - \frac{5.23}{100} - \frac{3.71}{100}\right) = 6756.7 \text{ KN}$$

$$\text{Equivalent upward UDL due to Cable Parabola} = \frac{8 \text{ x p x sag}}{L^2} = \frac{8 \text{ x } 6756.7 \text{ x } 0.095}{73^2}$$
$$= 0.96 \text{ KN/m}$$

Upward deflection due to PS-II transfer,

$$\frac{5wL^4}{384 \text{ x Eci x I}} = \frac{5 \text{ x } 0.96 \text{ x } 73^4}{384 \text{ x } 30358 \text{ x } 2.59} = 4.5 \text{ mm}$$

Deflection at Girder End eccentricity,

$$\frac{\text{p x } e_{\text{end x}} L^2}{8 \text{ x Eci x I}} = \frac{6756.7 \text{ x } 1.572 \text{ x } 73^2}{8 \text{ x } 30358 \text{ x } 2.59} = 90.3 \text{ mm}$$

d) Net Deflection = (Net Hogging – Net Sagging)

= [D_{PS-I} + D_{EPS-I} + D_{PS-II} + D_{EPS-II} – D_W]

= [253.5 – 27.3 + 4.5 + 90.3 – 268]

= 53 mm

3. DESIGN EXAMPLE OF A TWO-WAY POST-TENSIONED SLAB

The following example illustrates the design methods presented in ACI 318-05, IBC 2003 and BNBC 2020. Unless otherwise noted, all referenced table, figure, and equation numbers are from these books. The example presented here is for Two-Way Post-Tensioned Design.

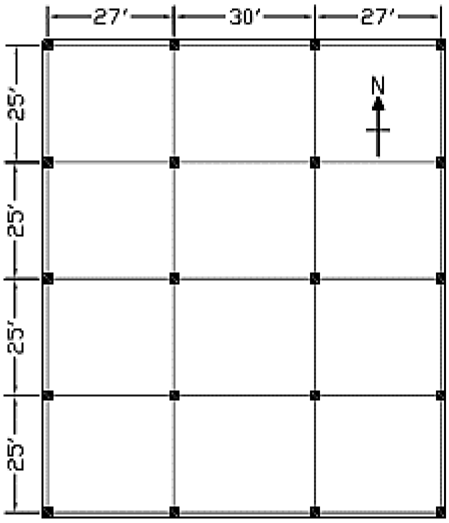


Figure 1: Typical Plan of a Slab

Loads:

Framing Dead Load = self-weight
Superimposed Dead Load = 25 psf [partitions and misc.]
Live Load = 40 psf
residential 2 hour fire-rating

Materials:

Concrete:

Normal weight 150 pcf
f'_c = 5,000 psi
f'_{ci} = 3,000 psi

Rebar:

f_y = 60,000 psi
PT: Unbonded tendons 1/2" ϕ, 7-wire strands, A = 0.153 in^2
f_{pu} = 270 ksi
Estimated prestress losses = 15 ksi (ACI 18.6)
f_{se} = 0.7 (270 ksi) - 15 ksi = 174 ksi (ACI 18.5.1)
P_{eff} = A*fse = (0.153) (174 ksi) = 26.6 kips/tendon

Determine Preliminary

Slab Thickness

Start with L/h = 45
Longest span = 30 ft h = (30 ft)(12)/45 = 8.0" preliminary slab thickness

Loading

$DL = \text{Selfweight} = (8in)\ (150\ pcf) = 100psf$

$SIDL = 25\ psf$

$LLo = 40\ psf$

Design of East-West Interior Frame

Use Equivalent Frame Method, ACI 13.7 (excluding sections 13.7.7.4-5)
Total bay width between centerlines = 25 ft
Ignore column stiffness in equations for simplicity of hand calculations
No pattern loading required, since LL/DL < 3/4 (ACI 13.7.6)

Calculate Section Properties

Two-way slab must be designed as Class U (ACI 18.3.3),
Gross cross-sectional properties allowed (ACI 18.3.4)

$A = bh = (300\ in)\ (8\ in) = 2{,}400\ in^2$

$S = bh^2/6 = (300\ in)\ (8\ in)^2/6 = 3{,}200\ in^3$

Set Design Parameters Allowable stresses:

Class U (ACI 18.3.3) At time of jacking (ACI 18.4.1) $f'_{ci} = 3{,}000$ psi

Compression $= 0.60\ f'_{ci} = 0.6(3{,}000\ psi) = 1{,}800$ psi

Tension $= 3\sqrt{f'_{ci}} = 3\sqrt{3{,}000} = 164$ psi

At service loads (ACI 18.4.2(a) and 18.3.3)

$f'_c = 5{,}000$ psi

Compression $= 0.45\ f'_c = 0.45(5{,}000\ psi) = 2{,}250$ psi

Tension $= 6\sqrt{f'_c} = 6\sqrt{5{,}000} = 424$ psi

Average precompression limits:

$P/A = 125$ psi min. (ACI 18.12.4)

300 psi max.

Target load balances:

60%-80% of DL (selfweight) for slabs (good approximation for hand calculation)
For this example: $0.75\ W_{DL} = 0.75(100\ psf) = 75\ psf$
Cover Requirements (2-hour fire rating, assume carbonate aggregate)
IBC 2003

Restrained slabs = 3/4″ bottom

Unrestrained slabs = 1 ½ ″ bottom

= 3/4″ top

Tendon profile:

Parabolic shape; For a layout with spans of similar length, the tendons will be typically be located at the highest allowable point at the interior columns, the lowest possible point at the mid-spans, and the neutral axis at the anchor locations. This provides the maximum drape for load-balancing.

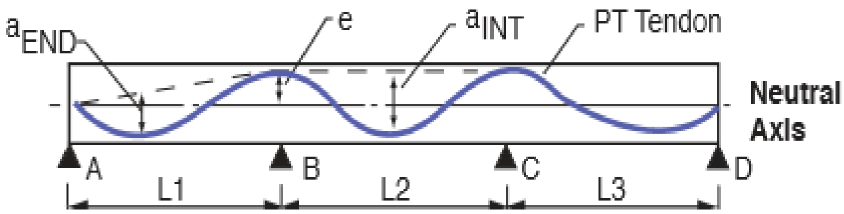


Figure 2: Tendon Profile

Tendon Ordinate	Tendon (CG) Location*
Exterior support – anchor	4.0”
Interior support – top	7.0”
Interior span – bottom	1.0”
End span – bottom	1.75”

(CG) = center of gravity
*Measure from bottom of slab

a_{INT} = 7.0” - 1.0” = 6.0”
a_{END} = (4.0” + 7.0”)/2 - 1.75” = 3.75”
eccentricity, e, is the distance from the center to tendon to the neutral axis; varies along the span

Prestress Force Required to Balance 75% of selfweight DL

Since the spans are of similar length, the end span will typically govern the maximum required post-tensioning force. This is due to the significantly reduced tendon drape, a_{END}.

$W_b = 0.75\ W_{DL}$
= 0.75 (100 psf) (25 ft)
= 1,875 plf
= 1.875 k/ft

For Exterior Span

Force needed in tendons to counteract the load in the end bay:

$P = W_b L^2 / 8a_{end}$
= (1.875 k/ft)(27 ft)^2 / [8(3.75 in / 12)]
= 547 k

Check Pre-Compression Allowance

Determine number of tendons to achieve 547 k

$\#$ tendons = (547 k) / (26.6 k/tendon)

= 20.56

Use 20 tendons

Actual force for banded tendons

P_{actual} = (20 tendons) (26.6 k) = 532 k

The balanced load for the end span is slightly adjusted

w_b = (532/547) (1.875 k/ft) = 1.82 k/ft

Determine actual Precompression stress

P_{actual} /A = (532 k)(1000) / (2,400 in^2)

= 221 psi > 125 psi min. ok

< 300 psi max. ok

Check Interior Span Force

P = (1.875 k/ft) (30 ft)2 / [8(6.0 in / 12)]

= 421 k < 532 k Less force is required in the center bay

For this example, continue the force required for the end spans into the interior span and check the amount of load that will be balanced:

w_b = (532 k) (8) (6.0 in /12) / (30 ft)^2

= 2.36 k/ft wb/wDL = 94%; [W_{DL}= 100*25=2.5 ksf]

This value is less than 100%; acceptable for this design.

East-West interior frame:

Effective prestress force, P_{eff} = 532 kips

Check Slab Stresses

Separately calculate the maximum positive and negative moments in the frame for the dead, live, and balancing loads. A combination of these values will determine the slab stresses at the time of stressing and at service loads.

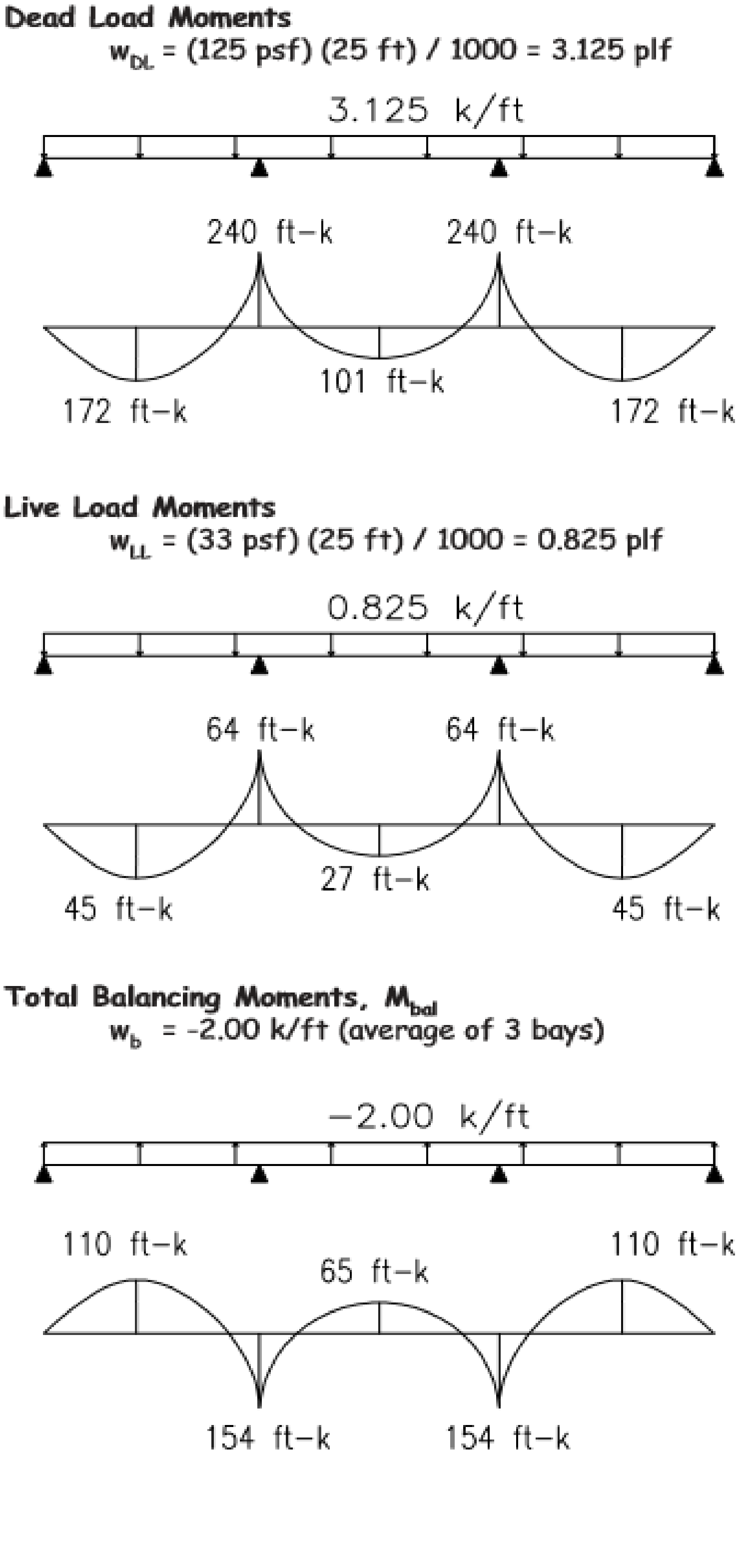


Figure 3: Moment Diagram for DL, LL and Balancing Load

Stage 1: Stresses immediately after jacking (DL + PT) (ACI 18.4.1)

Midspan Stresses

$f_{top} = (-M_{DL} + M_b)/S - P/A$

$f_{bot} = (+M_{DL} - M_b)/S - P/A$

Interior Span

$f_{top} = [(-101\text{ft-k} + 65\text{ft-k})(12)(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= -135 - 221 = -356$ psi compression $< 0.60\ f'_{ci} = 1800$ psi ok

$f_{bot} = [(101\text{ft-k} - 65\text{ft-k})(12)(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= 135 - 221 = -86$ psi compression $< 0.60\ f'_{ci} = 1800$ psi ok

End Span

$f_{top} = [(-172\text{ft-k} + 110\text{ft-k})(12)(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= -232 - 221 = -453$ psi compression $< 0.60\ f'_{ci} = 1800$ psi ok

$f_{bot} = [(172\text{ft-k} - 110\text{ft-k})(12)(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= 232 - 221 = 11$ psi tension $< 3\sqrt{f_{ci}} = 164$ psi ok

Support Stresses

$f_{top} = (+M_{DL} - M_b)/S - P/A$

$f_{bot} = (-M_{DL} + M_b)/S - P/A$

$f_{top} = [(240\text{ft-k} - 154\text{ft-k})(12)(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= 323 - 221 = 102$ psi tension $< 3\sqrt{f'_{ci}} = 164$ psi ok

$f_{bot} = [(-240\text{ft-k} + 154\text{ft-k})(12)(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= -323 - 221 = -544$ psi compression $< 0.60\ f'ci = 1800$ psi ok

Stage 2: Stresses at service load (DL + LL + PT) (18.3.3 and 18.4.2)

Midspan Stresses

$f_{top} = (-M_{DL} - M_{LL} + M_b)/S - P/A$

$f_{bot} = (+M_{DL} + M_{LL} - M_b)/S - P/A$

Interior Span

$f_{top} = [(-101\text{ft-k} - 27\text{ft-k} + 65\text{ft-k})(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= -236 - 221 = -457$ psi compression $< 0.45\ f'_c = 2250$ psi ok

$f_{bot} = [(101\text{ft-k} + 27\text{ft-k} - 65\text{ft-k})(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= 236 - 221 = 15$ psi tension $< 6\sqrt{f'_c} = 424$ psi ok

End Span

$f_{top} = [(-172\text{ft-k} - 45\text{ft-k} + 110\text{ft-k})(12)(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= -401 - 221 = -622$ psi compression $< 0.45\ f'_c = 2250$ psi ok

$f_{bot} = [(172\text{ft-k} + 45\text{ft-k} - 110\text{ft-k})(12)(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= 401 - 221 = 180$ psi tension $< 6\sqrt{f'_c} = 424$ psi ok

Support Stresses

$f_{top} = (+M_{DL} + M_{LL} - M_b)/S - P/A$

$f_{bot} = (-M_{DL} - M_{LL} + M_b)/S - P/A$

f_{top} = [(240ft-k + 64ft-k - 154ft-k)(12)(1000)]/(3200 in^3) - 221psi
= 563 - 221 = 342 psi tension < $6\sqrt{f'_c}$ = 424 psi ok
f_{bot} = [(-240ft-k - 64 ft-k + 154ft-k)(12)(1000)]/(3200 in^3) - 221psi
= -563 - 221 = -784 psi compression < 0.45 f'_c = 2250 psi ok
All stresses are within the permissible code limits.

Ultimate Strength

Determine factored moments
The primary post-tensioning moments, M_1, vary along the length of the span.
M_1 = P * e
e = 0 in. at the exterior support
e = 3.0 in at the interior support (neutral axis to the center of tendon)
M_1 = (532k) (3.0in) / (12) = 133ft-k

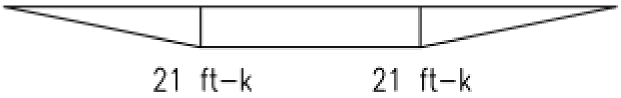


Figure 4: Secondary Moment Diagram

The secondary post-tensioning moments, M_{sec}, vary linearly between supports.
$M_{sec} = M_b - M_1$ = 154 ft-k - 133 ft-k
= 21 ft-k at the interior supports
The typical load combination for ultimate strength design is
$M_u = 1.2\ M_{DL} + 1.6\ M_{LL} + 1.0\ M_{sec}$

At midspan:

M_u = 1.2 (172ft-k) + 1.6 (45ft-k) + 1.0 (10.5 ft-k) = 289 ft-k

At support:

M_u = 1.2 (-240ft-k) + 1.6 (-64ft-k) + 1.0 (21 ft-k) = -370 ft-k

Determine minimum bonded reinforcement:

To see if acceptable for ultimate strength design.

Positive moment region:

Interior span: f_t = 15 psi < $2\sqrt{f'c} = 2\sqrt{5,000}$ = 141 psi
No positive reinforcement required (ACI 18.9.3.1)
Exterior span: f_t = 180 psi > $2\sqrt{f'c} = 2\sqrt{5,000}$ = 141 psi

Minimum positive moment reinforcement required
(ACI 18.9.3.2) / (BNBC 2020- Section 9.9.4)

$y = f_t/(f_t + f_c)h$
= [(180)/(180+622)](8 in)
= 1.80 in
$N_c = M_{DL+LL}/S * 0.5 * y * l2$
= [(172 ft-k + 45 ft-k)(12) / (3,200 in^3)](0.5)(1.80 in)(25ft)(12) = 220 k

$A_{s,\ min} = N_c / 0.5fy$ = (220 k) / [0.5(60ksi)] = 7.33 in^2

Distribute the positive moment reinforcement uniformly across the slab-beam width and as close as practicable to the extreme tension fiber.

$A_{s,\ min}$ = (7.33 in2)/(25 ft) = 0.293in^2/ft

Use #5 @ 12 in. oc Bottom = 0.31 in^2/ft (or equivalent)

Minimum length shall be 1/3 clear span and centered in positive moment region (ACI 18.9.4.1)

Negative moment region:

$A_{s,\ min} = 0.00075A_{cf}$ (ACI 18.9.3.3)

Interior supports:

A_{cf} = max. (8in) [(30ft + 27ft)/2, 25ft] *12

$As_{,\ min}$ = 0.00075(2,736 in2) = 2.05 in^2 = 11 - #4 Top (2.20 in^2)

Exterior supports:

A_{cf} = max. (8in) [(27ft/2), 25ft] *12

$A_{s,\ min}$ = 0.00075(2,400 in^2) = 1.80 in^2

= 9 - #4 Top (1.80 in2)

Must span a minimum of 1/6 the clear span on each side of support (ACI 18.9.4.2)

At least 4 bars required in each direction (ACI 18.9.3.3)

Place top bars within 1.5h away from the face of the support on each side (ACI 18.9.3.3)

= 1.5 (8 in) = 12 in

Maximum bar spacing is 12" (ACI 18.9.3.3)

Check minimum reinforcement if it is sufficient for ultimate strength

$M_n = (A_sf_y + A_{ps}f_{ps})$ (d-a/2)

d = effective depth

A_{ps} = 0.153in^2*(number of tendons) = 0.153in^2*(20 tendons) = 3.06 in^2

$f_{ps} = f_{se}$ + 10,000 + (f'_cbd)/(300A_{ps}) for slabs with L/h > 35 (ACI 18.7.2)

= 174,000psi + 10,000 + [(5,000psi) (25ft*12) d]/ [(300) (3.06 in^2)]

= 184,000psi + 1634d

$a = (A_sf_y + A_{ps}f_{ps}) / (0.85f'_cb)$

At supports d = 8" - 3/4" - 1/4" = 7"

f_{ps} = 184,000psi + 1634(7") = 195,438psi

a = [(2.20 in^2) (60 ksi) + (3.06 in^2) (195ksi)]/ [(0.85) (5ksi) (25ft*12)] = 0.57

ϕMn = 0.9 [(2.20 in^2) (60 ksi) + (3.06 in^2) (195ksi)] [7" - (0.57)/2]/12

= 0.9 (728k) (6.72in)/12 = 367 ft-k < 370ft-k

Reinforcement for ultimate strength requirements governs as, $_{reqd}$= 2.30in^2

12 - #4 Top at interior supports 9 - #4 Top at exterior supports

When reinforcement is provided to meet ultimate strength requirements, the minimum lengths must also conform to the provision of ACI 318-05 Chapter 12. (ACI 18.9.4.3)

At midspan (end span)

$d = 8" - 11/2" - 1/4" = 6\ 1/4"$

$fps = 184{,}000psi + 1634(6.25")$

$= 194{,}212psi$

$a= [(7.33\ in2)(60\ ksi) + (3.06\ in^2)(194ksi)]/[(0.85)(5ksi)(25ft*12)] = 0.81$

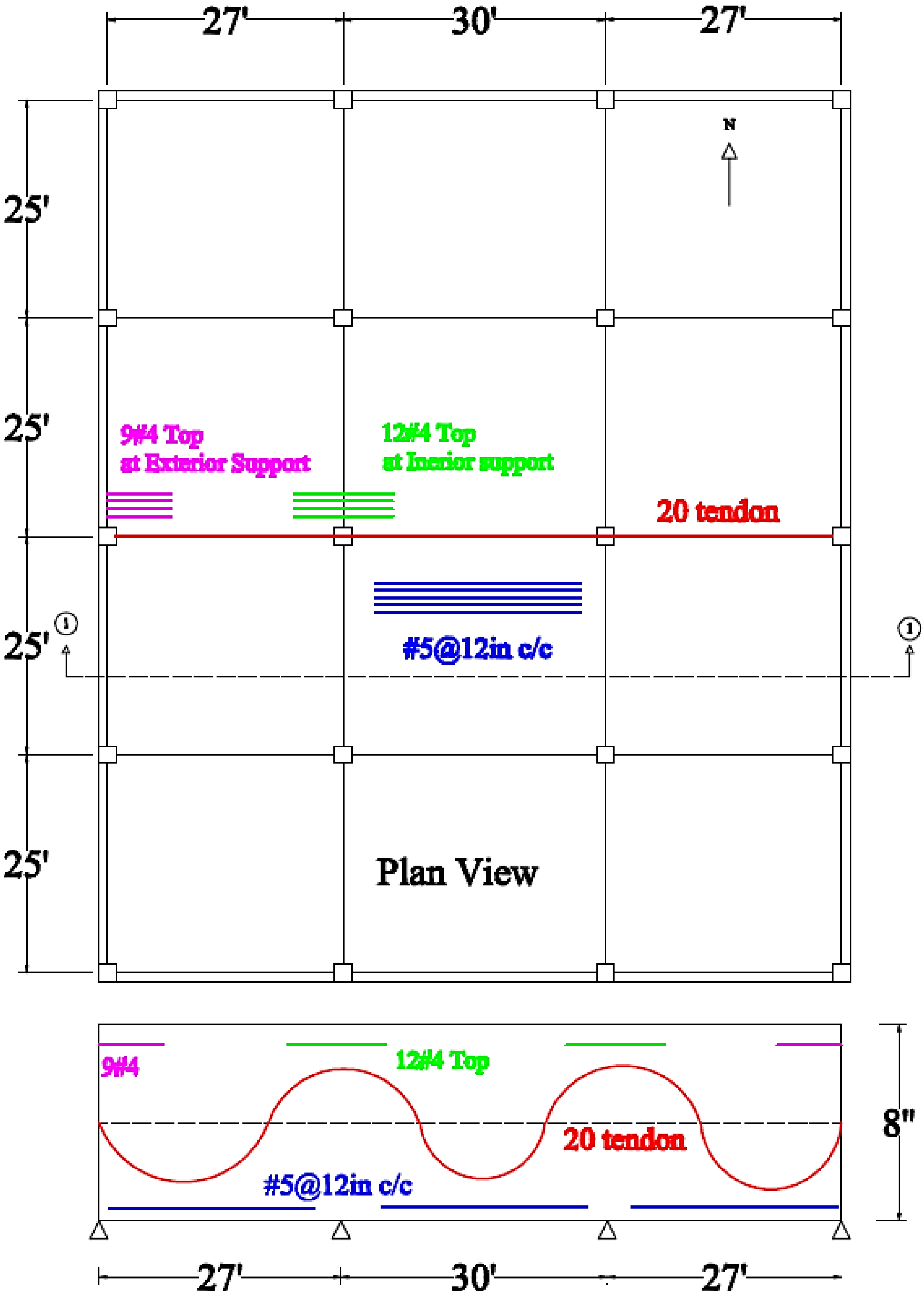
$\phi Mn = 0.9\ [(7.33\ in^2)(60\ ksi) + (3.06\ in^2)(194ksi)][6.25" - (0.81)/2]/12$

$= 0.9\ (1033k)(5.85in)/12 = 453\ ft\text{-}k > 289\ ft\text{-}k$

Minimum reinforcement ok

#5 @ 12"c/c Bottom at end spans

This is a simplified hand calculation for a post-tensioned two-way plate design. A detailed example can be found in the PCA Notes on ACI 318-05 Building Code Requirements for Structural Concrete. A simplified detailing of prestressed slab given below.



Section: 1-1

Figure- Detailing of Two way Post Tensioned Slab (E-W Direction)

4.Circular prestressing

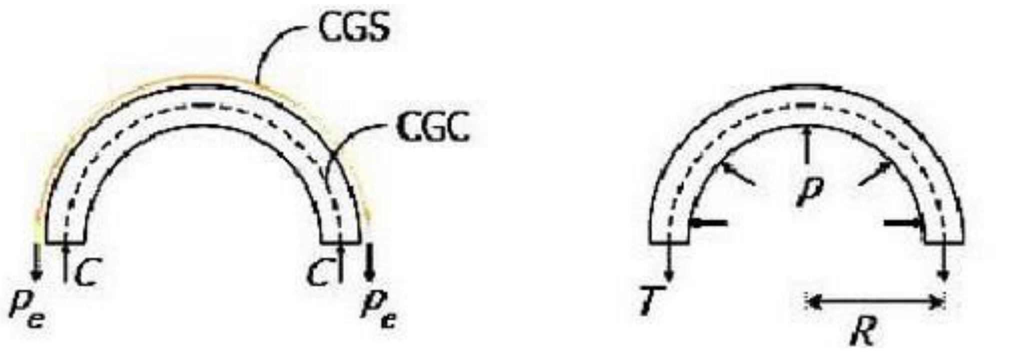
General

Circular Prestressing" is employed to denote the prestressing of circular structures such as pipes and tanks where the prestressing wires are wound in circles. In contrast to this term, "linear prestressing" is used to include all other types of prestressing, where the cables may be either straight or curved, but not wound in circles around a circular structure. In most prestressed circular structures, prestress is applied both circumferentially and longitudinally, the circumferential prestress being circular and the longitudinal prestress actually linear.
The circumferential prestressing resists the hoop tension generated due to the internal pressure. The prestressing is done by wires or tendons placed spirally, or over sectors of the circumference of the member. The wires or tendons lay outside the concrete core. Hence, the centre of the prestressing steel (CGS) is outside the core concrete section. When the prestressed members are curved, in the direction of prestressing, the prestressing is called circular prestressing. For example, circumferential prestressing in pipes, tanks, silos, containment structures and similar structures is a type of circular prestressing. In these structures, there can be prestressing in the longitudinal direction (parallel to axis) as well. Circular prestressing is also applied in domes and shells. [https://theconstructor.org]

Introduction

When the prestressed members are curved, in the direction of prestressing, the prestressing is called circular prestressing. For example, circumferential prestressing in pipes, tanks, silos, containment structures and similar structures is a type of circular prestressing. In these structures, there can be prestressing in the longitudinal direction (parallel to axis) as well. Circular prestressing is also applied in domes and shells. The circumferential prestressing resists the hoop tension generated due to the internal pressure. The prestressing is done by wires or tendons placed spirally, or over sectors of the circumference of the member. The wires or tendons lay outside the concrete core. Hence, the centre of the prestressing steel (CGS) is outside the core concrete section. The hoop compression generated is considered to be uniform across the thickness of a thin shell. Hence, the pressure line (or C-line) lies at the centre of the core concrete section (CGC). The following sketch shows the internal forces under service conditions. The analysis is done for a slice of unit length along the longitudinal direction (parallel to axis).

Liquid retaining structures, such as circular pipes, tanks and pressure vessels are admirably suited for circular prestressing. The circumferential hoop compression induced in concrete by prestressing counterbalances the hoop tension developed due to the internal fluid pressure. A reinforced concrete pressure pipe requires a large amount of reinforcement to ensure low-tensile stresses resulting in a crack-free structure. However, circular prestressing eliminates cracks and provides for an economical use of materials. In addition, prestressing safeguards against shrinkage cracks in liquid retaining structures. [https://www.scribd.com/document]



(a) Due to prestress

(b) Due to internal pressure

Fig1: Internal forces under service conditions
[https://www.scribd.com/document]

To reduce the loss of prestress due to friction, the prestressing can be done over sectors of the circumference. Buttresses are usedfor the anchorage of the tendons. The following sketch shows the buttresses along the circumference.

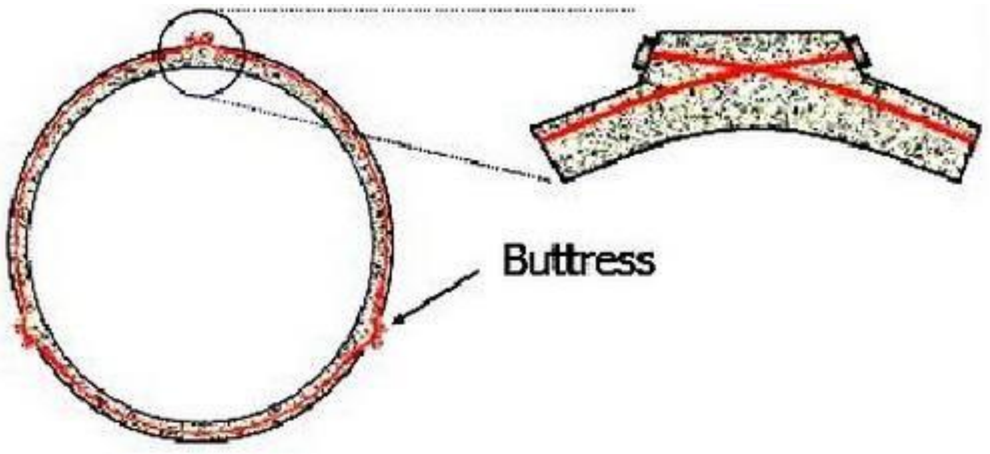


Fig 2: Use of buttress in Circular prestressing
[https://www.scribd.com/document]

Design parameters

General analysis

a) Analysis at transfer

The compressive stress can be calculated from the compression C. From equilibrium, C=Po, where P_o is the prestress at transfer after short-term losses. The compressive stress (f_c) i s giv e n as $f_c = -P_o /A$,
Where,
A=area of the longitudinal section of the slice. The permissible prestress is determined based on f_c within the allowable stress at transfer (f_{cc}/all).

b) Analysis at service loads

The tensile stress due to the internal pressure (p) can be calculated from the tension T. From equilibrium of half of the slice, T=ρR. Where, R is the radius of the mid-surface of the cylinder. The resultant stress (f_c) due to the effective prestress (P_e) and internal pressure is given as, $f_c = -P_o /A + \rho R/A_t$
A_t = area of the transformed longitudinal section of the slice.

Design

The internal pressure ρ and the radius are given variables. It is assumed that the prestressing steel alone carries the hoop tension due to internal pressure, that is,
$P_e = A_p f_{ps} = \rho R$.
The steps of design are as follows:
1) Calculate the area of the prestressing steel from the equation, $A_p = \rho R/f_{pe}$
2) Calculate the prestress at transfer from an estimate of the permissible initial stress f_{po} and using the equation $P_o = A_p f_{po}$.
3) Calculate the thickness of the concrete shell from the equation, A= Po/f_{cc}, here $f_{cc,all}$ is the allowable compressive stress at transfer.
4) Calculate the resultant stress f_c at the service condition. The value of f_c should be within $f_{cc,all}$ the allowable compressive stress at service conditions.

1) Design Example of a Post-Tensioned Composite Bridge Girder

ACI-ASCE Committee 343, “Analysis and Design of Reinforced Concrete Bridge Structures.” ACI Manual of Concrete Practice, Part 4, American Concrete Institute, Detroit, MI, 1989.

ASTM. (2006). “Standard specification for steel strand, uncoated seven-wire for pre-stressed concrete.” A416/A416M-06, West Conshohocken, PA.

AASTHO (2012), “AASTHO LRFD Bridge Design Specifications.” ISBN: 978-1-56051-523-4

2) DESIGN EXAMPLE OF A TWO-WAY POST-TENSIONED SLAB

Bangladesh National Building Code (BNBC) 2020.

Building code requirement for Structural Concrete, ACI 318-05 American Concrete Institute, 2005. [archive.org/stream/gov.law.aci.318.1995/aci.318.1995_djvu.txt]

Seismic Design of Precast Concrete Building Structures, IBC-2003 (International Building Code 2003).

3) Circular prestressing

[https://theconstructor.org]; [https://www.scribd.com/document]